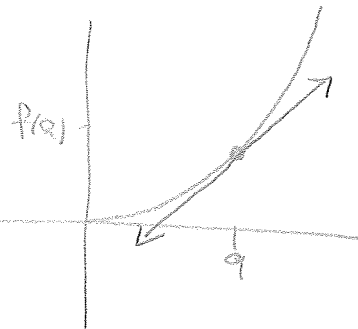


### § 3.11 - Linear Approximation

Goal: we want to approximate  $f(x)$  near the point  $(a, f(a))$

near  $(a, f(a))$   $f(x)$  is approximately the tangent line, so



$$f(x) \approx \text{tangent line} \leftarrow y - y_1 = m(x - x_1)$$

$$f(x) \approx y_1 + m(x - x_1)$$

$$f(x) \approx f(a) + f'(a)(x - a)$$

Example 1: (i) Find the linear approximation of  $f(x) = \sqrt{x+3}$  at  $a=6$ .

(ii) use it to find an approximate value to  $\sqrt{8}$ ,  $\sqrt{8.9}$ ,  $\sqrt{9.1}$ .

Solution:  $f'(x) = \frac{1}{2\sqrt{x+3}} \rightarrow f'(6) = \frac{1}{2\sqrt{6+3}} = \frac{1}{6}$

(i)  $f(x) \approx f(a) + f'(a)(x-a)$ ,  $f$

$$\approx \sqrt{6+3} + \frac{1}{6}(x-6) \approx 3 + \frac{1}{6}(x-6)$$

(ii)  $\sqrt{8} = \sqrt{5+3} = f(5) = 3 + \frac{1}{6}(5-6) = 2.8333 \dots$

$$\sqrt{8.9} = \sqrt{5.9+3} = f(5.9) = 3 + \frac{1}{6}(5.9-6) = 2.8933 \dots$$

$$\sqrt{9.1} = \sqrt{6.1+3} = f(6.1) = 3 + \frac{1}{6}(6.1-6) = 3.0166 \dots$$

Exercise 1: Use linear approximation to estimate  $\sin 0.8$ .

Exercise 2: Use linear approximation to estimate  $\sqrt[3]{65}$ .

