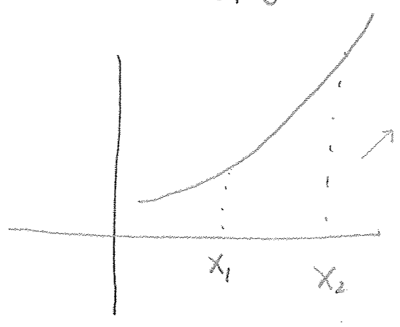
§ 4.3 - Monotonic Functions and first Derivative test

Increasing function

Algebra

if $x_1 \leq x_2$, then $f(x_1) \leq f(x_2)$

Geometry

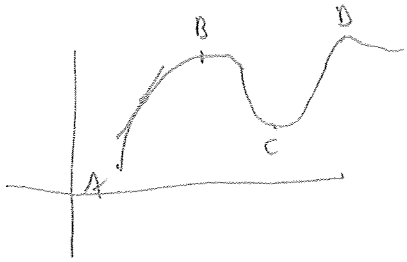


Exercise 1: write a similar definition for decreasing function.

Definition: A monotone function is either an increasing function or decreasing function.

Question: How to tell when a function is increasing/decreasing?

Answer: One way is from the definition which is hard to do in general. The other way is as follows



Slope of tangent line ≥ 0 we get increasing.

- If $f'(x) \geq 0$, then $f(x)$ is increasing.
- If $f'(x) \leq 0$, then $f(x)$ is decreasing.

Example 1: Find where the function $f(x) = 2x^3 + 3x^2 - 36x$ increasing or decreasing.

Solution: First we find the critical points, so we solve

$$f'(x) = 0$$

$$6x^2 + 6x - 36 = 0$$

$$x^2 + x - 6 = 0$$

$$\boxed{x = 2} \quad \text{or} \quad \boxed{x = -3}$$

-3 2

Sign of $f'(x)$: + + + + 0 − − − − − − 0 + + + +

local max   local min

increasing $(-\infty, 3)$ decreasing $(-3, 2)$ increasing $(2, \infty)$.

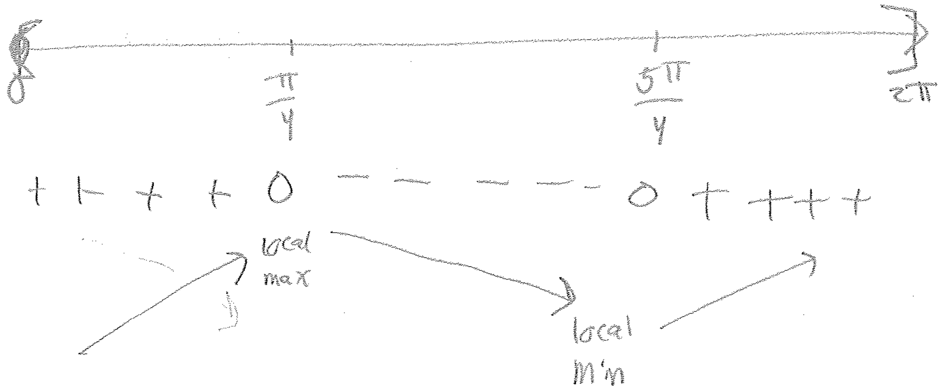
Example 2: Find where the function $f(x) = \sin x + \cos x$, $[0, 2\pi]$ is increasing and decreasing and find the local max/min.

Solution: $f'(x) = \cos x - \sin x$

we find the critical points as follows, we solve

$$f'(x) = \cos x - \sin x = 0$$

$$\cos x = \sin x \Rightarrow x = \frac{\pi}{4} \quad \text{or} \quad x = \frac{5\pi}{4}$$



increasing $\left(0, \frac{\pi}{4}\right)$

decreasing $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$

increasing $\left(\frac{5\pi}{4}, 2\pi\right)$.

c	$f(c)$
0	1
2π	1
$\frac{\pi}{4}$	$\sqrt{2}$ ← global max
$f(\frac{5\pi}{4}$	$-\sqrt{2}$ ↑ global min.

<u>Exercise 2</u>: Same as above but for $f(\theta) = 6\theta - \theta^2$.