§4.8 - Anti-Derivative

So far, we are given $f(x) \rightsquigarrow f'(x)$, $F(x) \leftsquigarrow f(x)$ such that $F'(x) = f(x)$

Goal: Given $f(x)$, Find $F(x)$ whose derivative is $f(x)$.

Such $F(x)$ is called anti-derivative.

Example 1: let $f(x) = x^2$

$F(x)$	$\frac{1}{3}x^3+2$	$\frac{1}{3}x^3+7$	...
$f(x)$	x^2		

$$\rightsquigarrow \underbrace{F(x) = \frac{1}{3}x^3 + C}_{\text{General anti-derivative}}$$

Example 2: Find the most general anti-derivative of the following functions

(a) $f(x) = \cos x$ (b) $f(x) = \frac{1}{x}$ (c) $f(x) = e^x$

(d) $f(x) = \sin x$ (e) $f(x) = x^n$ $(n \neq -1)$

Solution:

(a) $F(x) = \sin x$ (because $F'(x) = (\sin x)' = \cos x = f(x)$)

(b) $F(x) = \ln x$ (because $F'(x) = \frac{d}{dx}(\ln x) = \frac{1}{x} = f(x)$)

(c) $F(x) = e^x$ (because $F'(x) = \frac{d}{dx}(e^x) = e^x = f(x)$)

(d) $F(x) = -\cos x$ (because $F'(x) = \frac{d}{dx}(-\cos x) = \sin x = f(x)$

(e) $F(x) = \frac{1}{n+1} x^{n+1}$ (because $F'(x) = \frac{d}{dx}\left(\frac{1}{n+1} x^{n+1}\right) = \frac{n+1}{n+1} x^{n} = x^{n} = f(x)$)

Example 2: Find $g(x)$ such that

$$g'(x) = 4\cos x + \frac{3x^{\cancel{9}\,8} - \sqrt{x}}{x} = c$$

Solution:

$$g'(x) = 4\cos x + 3x^{8} - x^{-\frac{1}{2}}$$

$$g(x) = -4\sin x + \frac{3}{9}x^{9} + 2x^{\frac{1}{2}} = -4\sin x + \frac{1}{3}x^{9} + 2\sqrt{x} + C$$

Exercise 1: Find f if $f'(x) = e^{x} + 20(1+x^{2})^{-1}$ and $f(0) = -2$

Exercise 2: (a) $f(x) = \sec\theta\tan\theta - 2e^{3\theta}$ (b) $f(x) = \frac{x^{5} + x^{2} + 2x}{x^{2}}$

(c) $f(x) = (x+1)(x-1)$ (d) $f(x) = 2\sqrt{x} + 6\cos x + e^{2}$

Example 3: A particle moves in a straight line and has acceleration given by $a(t) = 3t + 4$. Its initial velocity is $v(0) = -6$ cm/s and initial position $s(0) = 3$ cm. Find the position function $s(t)$.

Solution: Recall $a(t) = v'(t)$

$$v(t) = \frac{3}{2}t^{2} + 4t + c \rightarrow v(0) = c = -6$$

$$v(t) = \frac{3}{2}t^{2} + 4t - 6$$

$$s(t) = \frac{3}{6}t^{3} + 2t^{2} - 6t + c_2 \longrightarrow s(0) = c_2 = 3$$

$$\boxed{s(t) = \frac{1}{2}t^{3} + 2t^{2} - 6t + 3}$$