§ 0.7 – Equations, in particular Linear Equations

① Equations

Q: What is an equation?

A: An equation is a statement that two expression are equal.

$$\underbrace{\text{expression } 1}_{\text{left side}} = \underbrace{\text{expression } 2}_{\text{right side}}$$

variable $x, y, z^2, \ldots$
numbers $3, \frac{1}{2}, \frac{3}{2}$
operations $(+, -, \times, \div, \sqrt{\ })$

Example 1: Equations

(a) $x + 2 = 3.$

(b) $x^2 + 3x + 7 = 5$

(c) $\dfrac{y}{y-4} = 6 + \dfrac{1}{3}$.

(d) $w = 7 - z$.

② Solution Set

Goal: To find the numbers (solution) of an equation.

Example 2:

(a) $x + 2 = 3$

→ $x = 1$ is a solution since $1 + 2 \boxed{=} 3$
$3 \boxed{=} 3$

→ $x = 3$ is not a solution, since $3 + 2 \boxed{\neq} 3$
$5 \boxed{\neq} 3$

$$\text{Solution set} = \{ \underbrace{1}_{\text{write all the solution}} \}$$

Example 3: check whether the following numbers are Solution or not:

Q4 $x^2+x-6=0$; $x=2,3$

Q6 $x(x^2+1)^2(x+2)=0$, $x=0,-1,2$.

3) Solving ~~Linear~~ Equations

A) Linear Equations

A linear equation in the variable x is an equation of the form

$$ax+b=0 \quad , \quad a\neq 0, \ b \text{ are constants.}$$

Example 4: Solve each of the following equations

(a) $5x-6=3x$

$5x-3x=6$

$\frac{2}{2}x=\frac{6}{2}$

$\boxed{x=3}$ → Solution set $=\{3\}$

Check

$5(3)-6 \stackrel{?}{=} 3(3)$

$15-6 \stackrel{?}{=} 9$

$9 \stackrel{?}{=} 9$ ✓

(b) $2(p+4)=7p+2$ --- open bracket method.

$2p+8=7p+2$

$8-2=7p-2p$

$6=5p$ → $\boxed{p=\frac{6}{5}}$

Solution set $=\{\frac{6}{5}\}$

(c) $\dfrac{7x+3}{2} - \dfrac{9x-8}{4} = 6$ --- ~~clear~~ clear the denominator method!

we multiply by everything in the denominator to clear it, so we multiply by 2 first to get

$$2\left(\frac{7x+3}{2}\right) - 2\left(\frac{9x-8}{\cancel{4}\,2}\right) = 2(6)$$

$$(7x+3) - \frac{(9x-8)}{2} = 12$$ --- multiply by 2 again.

$$2(7x+3) - 2\left(\frac{(9x-8)}{2}\right) = 2(12)$$

$$14x+6-(9x-8) = 24$$

$$14x+6-9x+8 = 24$$

$$5x+14 = 24 \longrightarrow 5x = 10 \rightarrow \boxed{x = \frac{10}{5} = 2}$$

Solution Set $= \{2\}$.

$\boxed{B}$ Literal Equation

Examples: Solve for each of the following:

(a) $I = prt$ for t, i.e., isolate t.

$$\frac{I}{pr} = \frac{\cancel{pr}t}{\cancel{pr}} \rightarrow \frac{I}{pr} = t$$

(b) $A = 2(l+w)$, for w.

$$\frac{A}{2} = l + w \longrightarrow \boxed{\frac{A}{2} - l = w}$$

C Fractional Equations

Example 6:

(a) $\frac{3}{x} + 5 = 2 \longrightarrow x\left(\frac{3}{x}\right) + x(5) = x(2)$

$3 + 5x = 2x \longrightarrow 3 = -3x$

$\boxed{x = -1}$

(b) $\frac{3x+4}{x+2} - \frac{3x-5}{x-4} = \frac{12}{x^2 - 2x - 8}$, let us first factor

$x^2 - 2x - 8 = (x+2)(x-4)$

4x

$\frac{(3x+4)}{(x+2)} - \frac{(3x-5)}{(x-4)} = \frac{12}{(x+2)(x-4)}$

$(3x+4)(x-4) - (3x-5)(x+2) = 12$

$\vdots$

$\boxed{x = -2}$

D Radical Expression

(a) $\sqrt{x+3} = 2$, to cancel the radical, take the square of both sides.

$(\sqrt{x+3})^2 = (2)^2$

$x + 3 = 4$

$\boxed{x = 1}$

$x - 2\sqrt{x(x+1)} + x + 1 = 1 \longrightarrow \ldots$

(b) $\sqrt{x} - \sqrt{x+1} = 1 \longrightarrow (\sqrt{x} - \sqrt{x+1})^2 = 1$