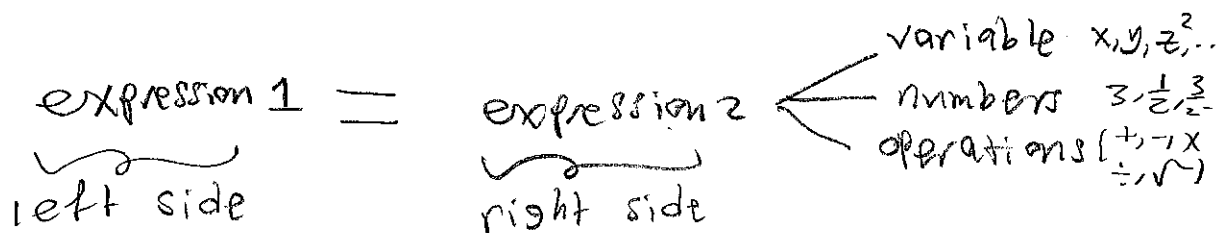


# § 0.7 - Equations, in particular Linear Equations

## ① Equations

Q: What is an equation?

A: An equation is a statement that two expressions are equal.



## Example 1: Equations

(a)  $x + z = 3$

(b)  $x^2 + 3x + 7 = 5$

(c)  $\frac{y}{y-4} = 6 + \frac{1}{3}$

(d)  $w = 7 - z$

## ② Solution Set

Goal: To find the numbers (solution) of an equation.

### Example 2:

(a)  $x + 2 = 3$   $\rightarrow$   $x = 1$  is a solution since  $1 + 2 \equiv 3$   
 $3 \equiv 3$

$\rightarrow$   $x = 3$  is not a solution, since  $3 + 2 \not\equiv 3$   
 $5 \not\equiv 3$

Solution set =  $\{ \underline{1} \}$

write all the solution

Example 3: check whether the following numbers are Solution or not.

Q4)  $x^2 + x - 6 = 0$  ;  $x = 2, 3$

Q6)  $x(x^2 + 1)^2(x + 2) = 0$  ,  $x = 0, -1, 2$ .

### 3) Solving Linear Equations

#### A) Linear Equations

A linear equation in the variable  $x$  is an equation of the form  $ax + b = 0$  ,  $a \neq 0$ ,  $b$  are constants.

Example 4: solve each of the following equations

(a)  $5x - 6 = 3x$

$$5x - 3x = 6$$

$$\frac{2}{2}x = \frac{6}{2}$$

$$x = 3$$

solution set =  $\{3\}$

check

$$5(3) - 6 \stackrel{?}{=} 3(3)$$

$$15 - 6 \stackrel{?}{=} 9$$

$$9 \stackrel{?}{=} 9 \checkmark$$

(b)  $2(p+4) = 7p + 2$  --- open bracket method.

$$2p + 8 = 7p + 2$$

$$8 - 2 = 7p - 2p$$

$$6 = 5p \longrightarrow$$

$$p = \frac{6}{5}$$

Solution set =  $\left\{ \frac{6}{5} \right\}$

$$(c) \frac{7x+3}{2} - \frac{9x-8}{4} = 6 \quad \text{--- clear the denominator method!}$$

we multiply by everything in the denominator to clear it, so we multiply by 2 first to get

$$2\left(\frac{7x+3}{2}\right) - 2\left(\frac{9x-8}{4}\right) = 2(6)$$

$$(7x+3) - \frac{(9x-8)}{2} = 12 \quad \text{--- multiply by 2 again}$$

$$2(7x+3) - 2\left(\frac{9x-8}{2}\right) = 2(12)$$

$$14x+6 - (9x-8) = 24$$

$$14x+6 - 9x+8 = 24$$

$$5x+14 = 24 \rightarrow 5x = 10 \rightarrow \boxed{x = \frac{10}{5} = 2}$$

Solution Set =  $\{2\}$

## B Literal Equation

Examples solve for each of the following:

(a)  $I = prt$  for  $t$ , i.e., isolate  $t$

$$\frac{I}{pr} = \frac{prt}{pr} \rightarrow \frac{I}{pr} = t$$

(b)  $A = 2(l+w)$ , for  $w$

$$\frac{A}{2} = l+w \rightarrow \boxed{\frac{A}{2} - l = w}$$

## C Fractional Equations

Example 6:

$$(a) \frac{3}{x} + 5 = 2 \rightarrow x\left(\frac{3}{x}\right) + x(5) = x(2)$$

$$3 + 5x = 2x \rightarrow 3 = -3x$$

$$\boxed{x = -1}$$

$$(b) \frac{3x+4}{x+2} - \frac{3x-5}{x-4} = \frac{12}{x^2-2x-8}$$

let us first factor

$$x^2 - 2x - 8 = (x+4)(x-4)$$

$$\frac{(3x+4)}{(x+2)} - \frac{(3x-5)}{(x-4)} = \frac{12}{(x+2)(x-4)}$$

$$(3x+4)(x-4) - (3x-5)(x+2) = 12$$

⋮

$$\boxed{x = -2}$$

## D Radical Expression

(a)  $\sqrt{x+3} = 4$ , to cancel the radical, take the square of both sides.

$$(\sqrt{x+3})^2 = (4)^2$$

$$x+3 = 4$$

$$\boxed{x = 1}$$

$$x - 2\sqrt{x(x+1)} + x + 1 = 1 \rightarrow \dots$$

$$(b) \sqrt{x} - \sqrt{x+1} = 1 \rightarrow (\sqrt{x} - \sqrt{x+1})^2 = 1$$