

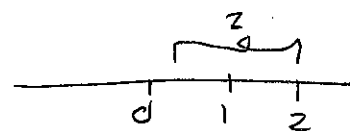
§ 1.4 - Absolute Value

Definition:

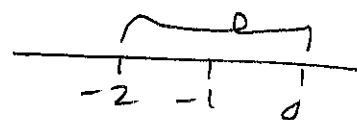
The absolute value of any number x is the distance from x to the zero. We denote it by $|x|$.

Example:

$$\begin{aligned} |2| &= \text{distance between } 2 \text{ \& } 0 \\ &= 2 \end{aligned}$$



$$\begin{aligned} |(-2)| &= \text{distance between } -2 \text{ \& } 0 \\ &= 2 \quad (-(-2)) \end{aligned}$$



$$|0| = 0$$

$$|x| = \begin{cases} x & , x \geq 0 \\ -x & , x < 0 \end{cases}$$

Properties of absolute values

$$1. |ab| = |a| \cdot |b|$$

$$4. -|a| \leq a \leq |a|$$

$$2. \left| \frac{a}{b} \right| = \frac{|a|}{|b|}$$

$$5. |a+b| \leq |a| + |b|$$

$$3. |a-b| = |b-a|$$

Absolute value equations and Inequalities

It is an equation (inequality) that involves variable x to be found.

Rules

1. $|X| = a \rightarrow X = a$ or $X = -a$
2. $|X| < a \rightarrow -a < X < a$
3. $|X| > a \rightarrow X > a$ or $X < -a$

Examples

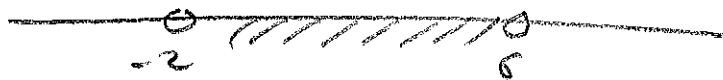
① $|X-3| = 2 \rightarrow X-3 = 2$ or $X-3 = -2$
 $X = 5$ or $X = 1$

Solution set = $\{5, 1\}$

② (exercise) $|7 - 3X| = 5$

③ $|X-4| = -3$ No solution as the distance cannot be negative.

④ $|X-2| < 4 \rightarrow -4 < X-2 < 4$
 $-4+2 < X < 4+2$
 $-2 < X < 6$



$(-2, 6)$

5 (exercise) $|3-2x| \leq 5$.

6 $|x+5| \leq -2$, No solution as the distance can't be negative!

7 $|x+5| \geq 7 \rightarrow x+5 \geq 7$ or $x+5 \leq -7$
 $x \geq 2$ or $x \leq -12$



$(-\infty, -12] \cup [2, \infty)$

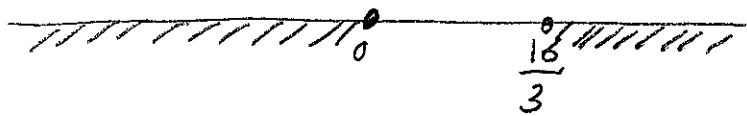
8 (exercise) $|3x-4| > 1$.

9 $|\frac{3x-8}{2}| \geq 4 \rightarrow \frac{3x-8}{2} \geq 4$ or $\frac{3x-8}{2} \leq -4$

$3x-8 \geq 8$ or $3x-8 \leq -8$

$3x \geq 16$ or $3x \leq 0$

$x \geq \frac{16}{3}$ or $x \leq 0$



$(-\infty, 0] \cup [\frac{16}{3}, \infty)$

