§2.1- Functions

1. Definition of a function.
2. Finding the domain of a function.
3. Finding Function values.
4. Application of functions.

1- Definition of a function

A function from a set X to a set Y is an assignment (rule) that tells how one element x in X is related to only one element y in Y.

Notation:

- $f: X \to Y$.
- $y = f(x)$ "f of x"
- x is the input (independent variable) and $y = f(x)$ is the output (dependent variable)
- The set X is called the domain (input) and the set Y is called the Co-domain. while the set of all output is called the range.

Question: How to describe a function? "by the formula"

Example 0.1:
$$f : \mathbb{R} \longrightarrow \mathbb{R}$$
$$x \mapsto 3x+1$$

or $f(x) = 3x+1$ ← - - - this is how we describe it.

- $f(1) = 3(1)+1 = 4$
- $f(0) = 3(0)+1 = 1$
- $f(-2) = 3(-2)+1 = -5$.
- $f(-7) = 3(-7)+1 = -20$ ...

Example 0.2:
$$f : \mathbb{R} \longrightarrow \mathbb{R}$$
$$x \mapsto x^2$$

$f(x) = x^2$

- $f(0) = 0$
- $f(1) = 1$
- $f(3) = 9$
- $f(-1) = 1$
- $f(-9) = 81$
- $f(-2) = 4$

Domain = $\mathbb{R}$

co-domain = $\mathbb{R}$ -- fixed

range $= \{y \mid y \geq 0\} = [0, \infty)$.

Example 0.3:
$$f : \mathbb{R} \longrightarrow \mathbb{R}$$
$$x \mapsto \frac{1}{x}$$

- $f(1) = \frac{1}{1} = 1$
- $f(-2) = \frac{1}{-2}$
- $f(0) = \frac{1}{0}$ ← problem, so we have to exclude 0 from the domain

domain $= \{x \mid x \neq 0\}$.

2- Finding the domain of functions

Recall:

Domain of f is the set of all x such that $f(x)$ make sense (i.e., No problems like zero in the denominator or negative inside even root, etc).

Domain of $f = \{ x \mid f(x) \text{ makes sense} \}$.

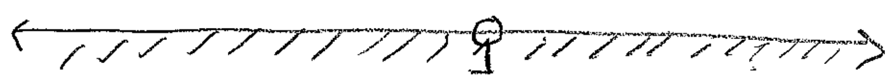
Example 1: Find the domain of $f(x) = \dfrac{3}{x-1}$.
(zero in denominator)

Here we would have problems only if the denominator is equal to zero, so we need to find those values and exclude them. So we solve

$$x - 1 = 0 \Rightarrow x = 1$$

So the domain of f is all the values except $x = 1$.

Domain of $f = \{ x \mid x \neq 1 \} =$



$(-\infty, 1) \cup (1, \infty)$.

Exercise: Find the domain of $f(x) = \dfrac{2x+1}{3x+8}$

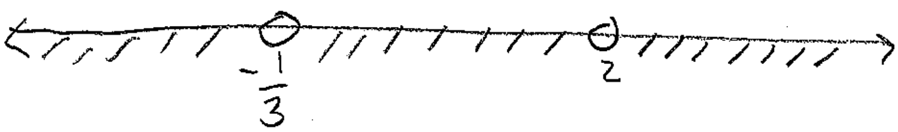
Example 2: Find the domain of $f(x) = \dfrac{x^2-1}{3x^2-5x-2}$

Similarly to the previous example, we find all the values that make the denominator equal to zero

$$3x^2 - 5x - 2 = 0$$

$$x = 2 \quad \text{or} \quad x = \frac{-1}{3} \quad \left(\S\, 0.8 \text{ by the formula}\right)$$

Domain of $f = \{ x \mid x \neq 2 \text{ and } x \neq -\frac{1}{3} \}$



$$(-\infty, -\tfrac{1}{3}) \cup (-\tfrac{1}{3}, 2) \cup (2, \infty)$$

Exercise $g(y) = \dfrac{1}{y^2+3}$.

Example 3 (Negative inside the root) Find the domain of $f(x) = \sqrt{2x-4}$.

Here we would have problem only if there is negative inside the square root, so we have to find all values that make $2x-4$ greater than or equal to 0, i.e.,

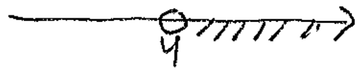
$$2x - 4 \geq 0 \to x \geq 2 \rightsquigarrow \text{Domain} = \{x \mid x \geq 2\}$$

$[2, \infty)$.

Example 4: $f(x) = \dfrac{3}{\sqrt{x-4}}$

Here we would have problem if we have negative in the root and zero in the denominator, so we find all the values that gives <u>only</u> positive values, so

$x - 4 > 0 \to x > 4 \leadsto$ Domain $= \{x \mid x > 4\}$



$(4, \infty)$.

<u>Exercise</u>: $f(x) = \dfrac{3x^2+7}{\sqrt{3x+6}}$.

3- <u>Finding Function Values</u>

<u>Recall</u>: $(a \pm b)^2 = a^2 \pm 2ab + b^2$

<u>Example 5</u>: let $g(x) = x^2 - 2$, Find

- $g(2) = (2)^2 - 2 = 4 - 2 = 2$

"we replace each x by 2"

- $g(u) = u^2 - 2$
- $g(u^2) = (u^2)^2 - 2 = u^4 - 2$
- $g(u+1) = (u+1)^2 - 2 = u^2 + 2u + 1 - 2 = u^2 + 2u - 1$.

Exercise let $f(x) = \dfrac{x-5}{x^2+3}$

- $f(5)$ · $f(2x)$ · $f(x+h)$ · $f(-7)$

Example 6: $f(x) = x^2+2x$

- $f(x+h) = (x+h)^2 + 2(x+h) = x^2+2xh+h^2+2x+2h$
- $\dfrac{f(x+h)-f(x)}{h} = \dfrac{x^2+2xh+h^2+2x+2h-(x^2+2x)}{h}$

$$= \frac{\cancel{x^2}+2xh+h^2+\cancel{2x}+2h-\cancel{x^2}-\cancel{2x}}{h}$$

$$= \frac{2xh+h^2+2h}{h} = \frac{h(2x+h+2)}{h}$$

$$= 2x+h+2.$$

Exercise: Find $\dfrac{f(x)-f(2)}{x-2}$ if $f(x) = 2x^2-x+1$.

4- Application of functions

Example 7 (Demand function)

$$p = \frac{120}{q}$$

price per unit

# of units

If the price is 60 per unit, how many units we have?

$$60 = \frac{120}{q} \longrightarrow q = \frac{120}{60} = 2$$

Example 8:

A company has capital of 7000 BD and weekly income of 320 BD and weekly expenses of 210 BD. Find the value V of the company in term of t = # of weeks.

$$V = 7000 + (320 - 210)\,t$$

$$\boxed{V = 7000 + 110t}$$

Exam Question 1 Find the domain

(a) $f(x) = 2x+5$ (b) $g(x) = \dfrac{4}{x^2-9}$ (c) $h(x)=\sqrt{3x+7}$

2 Demand function $p = \dfrac{165}{q} + 4$, Find the least number of X units to get revenue