§2.2- Special Functions

Example 1: (Constant function) let c be a constant.

$$f : \mathbb{R} \longrightarrow \mathbb{R}$$
$$x \longmapsto 3$$

So

- $f(x) = 3$
- $f(0) = 3$
- $f(1) = 3$
- $f(4) = 3$
- $f(x+1) = 3$

... So the output is always 3

So range $= \{ y \mid y = 3 \} = \{3\}$.

So in general, $f(x) = c$, where c is a constant is called constant function.

(recall linear and quadratic equations)

Example 2: (Polynomial function)

A polynomial function is a function of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$$

called coefficient ✓ and is called the leading coefficient

where $a_n, a_{n-1}, \dots, a_0$ are real numbers ✓ with $a_n \neq 0$ ✓ and $n \geq 0$ is an integer. and is called the degree of $f(x)$ and

Example 3:

$f(x)$	Degree	leading coefficien
$3x+1$	1	3
x^2+x+1	2	1
$1+2x+2x^3+x^2$	3	2
7	0	7
$\frac{x^2+5x+10}{7}$	2	$\frac{1}{7}$

$3x+1$ -- linear function

x^2+x+1 --- quadratic function.

Example 4: (Non-polynomial Example)

- $f(x) = \frac{x^2+1}{x}$ --- rational function.
- $f(x) = \sqrt{x}$
- $f(x) = 3^x$ ..

$\vdots$

Example 5 (Rational function)

A rational function is a quotient of two polynomials ("polynomial over polynomial")

~~Exa~~ • $f(x) = \frac{x^2-2x}{x+1}$, $f(x) = \frac{2x+3}{x-1}$. $f(x) = x^{-4} = \frac{1}{x^4}$.

Example 6: Case - Defined Functions

$\boxed{1}$ $$g(x) = \begin{cases} x-1, & x \geq 3 \\ 3-x^2, & x < 3 \end{cases}$$

$\cdot g(1)$ $\cdot g(2)$ $\cdot g(4)$ $\cdot g(6)$, $\cdot g(3)$

$\boxed{2}$ $$F(t) = \begin{cases} 2, & t > 1 \\ 0, & t = 1 \\ -1, & t < 1 \end{cases}$$

$F(12)$, $F(-\sqrt{3})$, $F(1)$, $F(\frac{15}{2})$