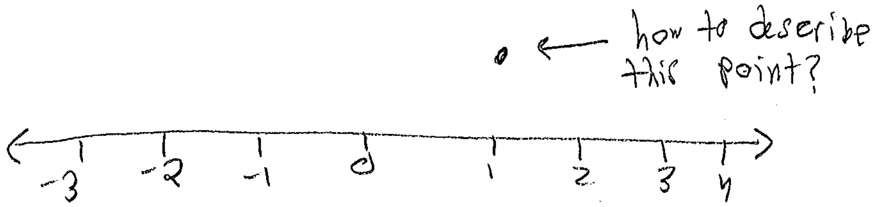
§2.5 - Graphs in Rectangular Coordinates

- Coordinate System
- Graphing function, intercepts, domain & range
- Reading information from the graph of the function.
- Horizontal and vertical Line tests.

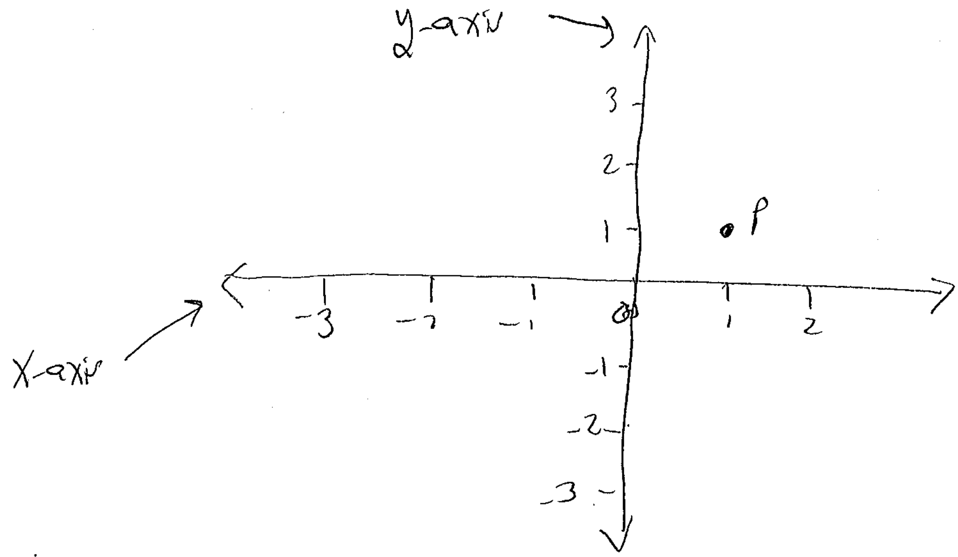
1- Rectangular Coordinate System

<u>Motivational Example</u>: Consider the number line



<u>Solution</u>:

we take another copy of the number line and we put it perpendicular to the old one so that their origins coincide.



P is described by an ordered pair $(1, 2)$ — x-coordinate, y-coordinate [So the order is important]

Special point is $(0, 0)$ which we call the <u>origin</u>.

So to locate any point, X-coordinate tells how many steps to go (left or right)

, y-coordinate tells us how many steps to go up and down.

Quadrant II	Quadrant I
Quadrant III	Quadrant IV

Example 1: Label each point and give the quadrant

$(-2,-5)$, $(3,-1)$, $(-\frac{1}{3}, 4)$, $(1,0)$, $(-4,5)$, $(3,0)$, $(0,-6)$

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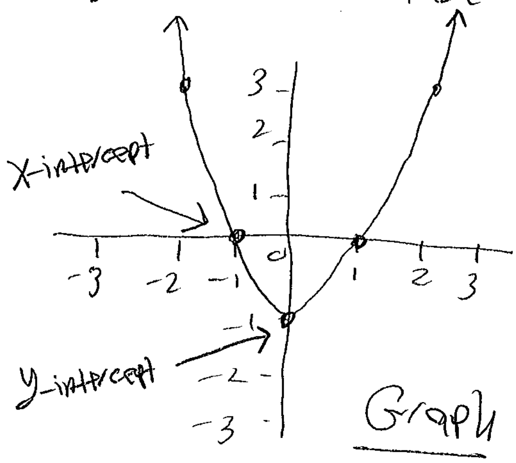
2. Graphs, intercepts, domain & range

Example 2: Graph the function $y = x^2 - 1$.

Solution:

we substitute values of x and we find y to fill the table

x	-2	-1	0	1	2	3
$y=f(x)$	3	0	-1	0	3	8



Graph

Note:

- In ideal world, we will need to plot infinitely many points to get a perfect graph, but this is <u>not</u> possible, so we concern only on the "general shape" by joining only serveral points by a smooth curve whenever possible.

- In MATHS 104, we will be able to graph more complicated functions in an easier way.

Definition:

- x-intercept is the point where the graph of the function intersects with x-axis.

- y-intercept is the point where the graph of the function intersects with y-axis.

To find, x-intercept, set $y=0$ and find x

. y-intercept, set $x=0$ and find y.

Example 3: Find the x-intercept, y-intercept, of the graph $y = 3x-6$ and sketch its graph.

Solution:

1- x-intercept: set $y=0$, so we get

$$0 = 3x - 6$$
$$6 = 3x \quad \rightarrow \quad x = 2$$
$$\frac{6}{3} = x$$

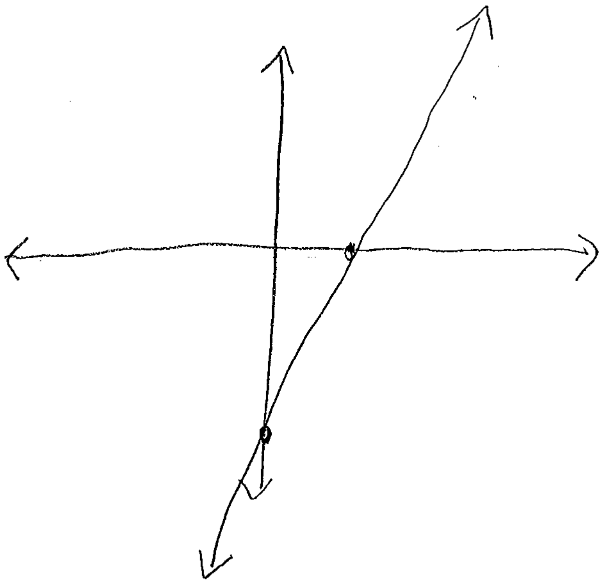
and the x-intercept is $\boxed{(\underbrace{2}_{x}, \underbrace{0}_{y})}$

2- y- intercept set $x=0$,

$$y = 3(0) - 6 = -6 \rightarrow \text{y-intercept is } (\underbrace{0}_{x}, \underbrace{-6}_{y})$$

3- Sketch plot some extra points and join them

x	0	2	1	-1
y	-6	0	-3	-9



Domain $= (-\infty, \infty)$

Range $= (-\infty, \infty)$

Exercise 2: $y = 3 - 2x$

Example 4: Find x-intercept, y-intercept, domain, range, and sketch

$$y = 4x^2 - 16$$

x-intercept: set $y = 0$, so

$$0 = 4x^2 - 16$$

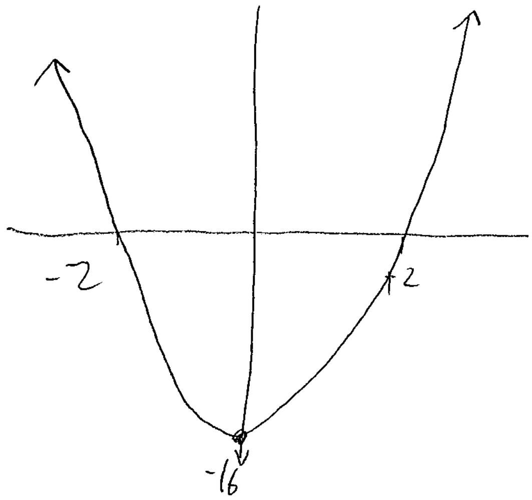
$$0 = 4(x^2 - 4) \rightarrow x = 2 \quad \text{or} \quad x = -2$$

$(2,0)$ and $(-2,0)$.

y-intercept: Set $x = 0$, so

$$y = 4(0)^2 - 16 = -16$$

So y-intercept is the point $(0, -16)$.



Domain $= (-\infty, \infty)$

Range $= [-16, \infty)$.

Example 5: $y = \dfrac{-1}{x}$

x-intercept: set $y = 0$,

$$0 = \frac{-1}{x}$$

$$0 \cdot x = \frac{-1}{x} \cdot x$$

x disappears

$$0 = -1 \quad \text{FALSE}$$

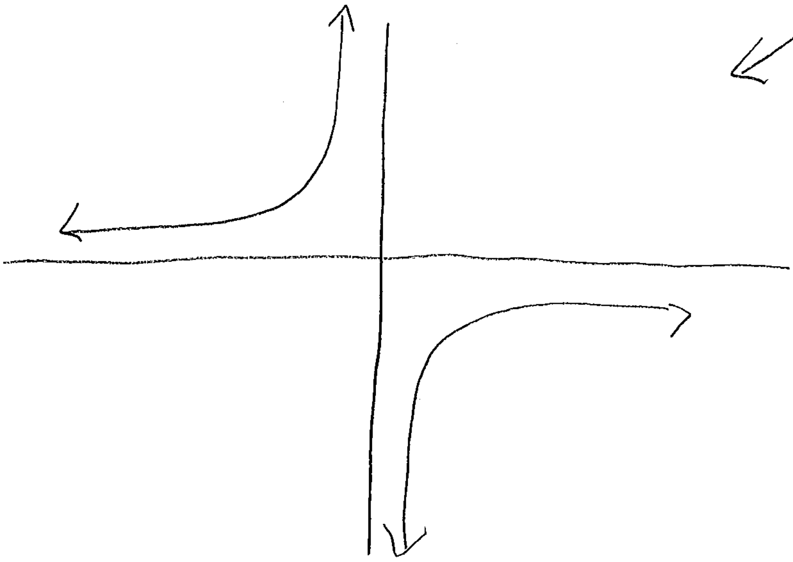
So No solution and so no x-intercept.

y-intercept: set $x = 0$

$$y = \frac{-1}{0} \quad \text{undefined}$$

so No solution and so no y-intercept, so we make a table

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{3}$	$\frac{1}{2}$	1	$-$	1	$\frac{-1}{2}$	$\frac{-1}{3}$

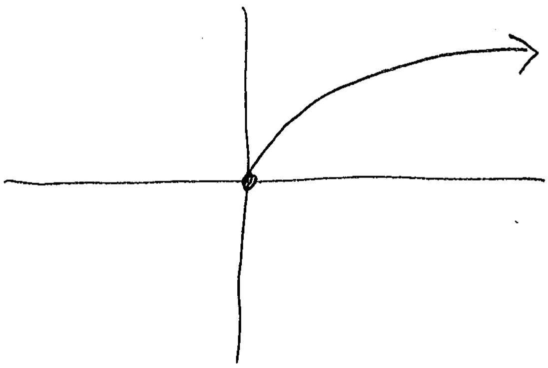


General shape for $\frac{1}{x}$.

Example 6: $y = \sqrt{x}$

X-intercept $y=0 \to x=0 \to (0,0)$

y-intercept $x=0 \to y=0 \to (0,0)$



General shape for $\sqrt{x}$

Domain $= [0, \infty)$

Range $= [0, \infty)$.

Example 7: (Exam Question) $y = \sqrt{3x+1}$

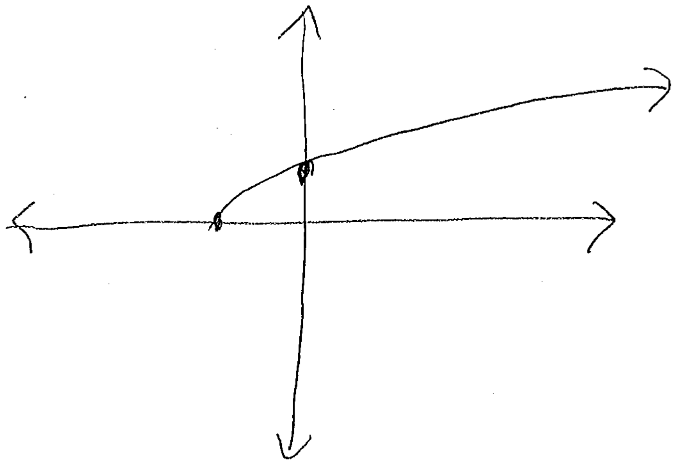
X-intercept: set $y=0 \to 0 = \sqrt{3x+1} \to x = -\frac{1}{3} \to \left(-\frac{1}{3}, 0\right)$

$0^2 = (\sqrt{3x+1})^2$

$0 = 3x+1$

y-intercept: set $x=0 \rightarrow y=\sqrt{3(0)+1} = \sqrt{1} = 1 \rightarrow (0,1)$

Plot some points and make a table.

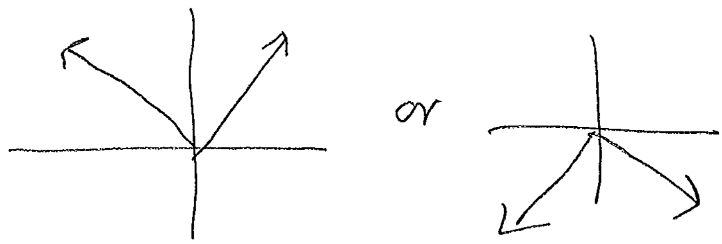


Domain $= [-\frac{1}{3}, \infty)$

Range $= [0, \infty)$.

Exercise: $y = \sqrt{x+1} - 2$

Example 8: $y = |x| \rightarrow$



Example 9: Sketch $y = |3x+2|$

x-intercept: set $y=0$

$$0 = |3x+2|$$

$$3x+2=0 \Longrightarrow \left(-\frac{2}{3}, 0\right)$$

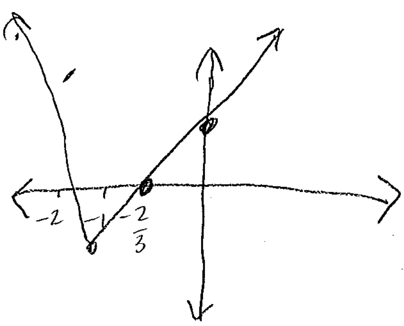
$$x = \frac{-2}{3}$$

y-intercept: set $x=0$

$$y = |3(0)+2| = |2| = 2$$

$(0,2)$

x	-3	-2	-1	0	1	2	3
y	7	4	-1	2	5	8	



Example 9: sketch $y = \sqrt{x^2-4}$

X-intercept $y=0$

$0 = \sqrt{x^2-4}$

$x = 2$ or $x = -2$

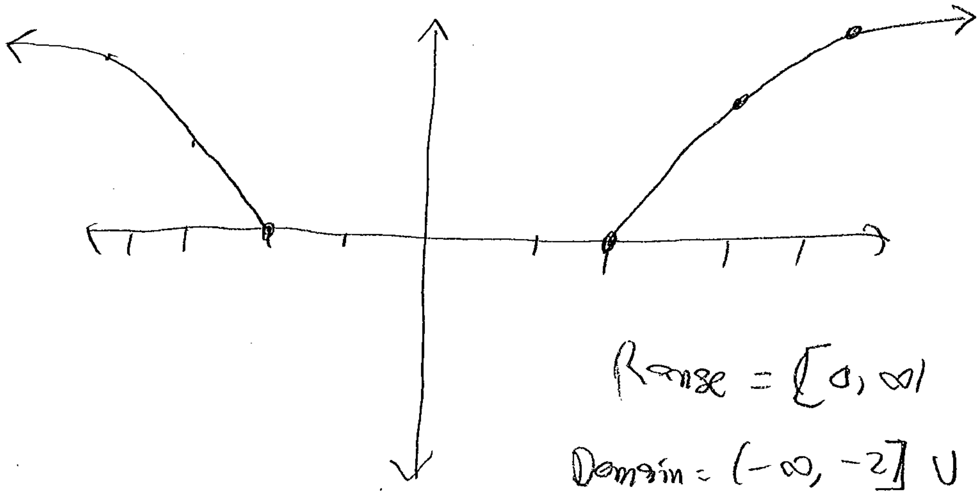
$(2,0)$ & $(-2,0)$

y-intercept $x=0$

$y = \sqrt{0^2-4} = \sqrt{-4}$!

No y-intercept.

x	-4	-3	-2	-1	0	1	2	3	4
y	12	5	0	x	x	x	0	5	12

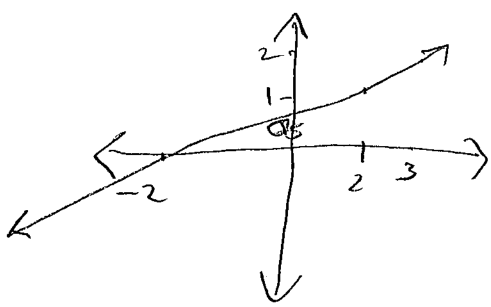


Range = $[0, \infty)$

Domain = $(-\infty, -2] \cup [2, \infty)$

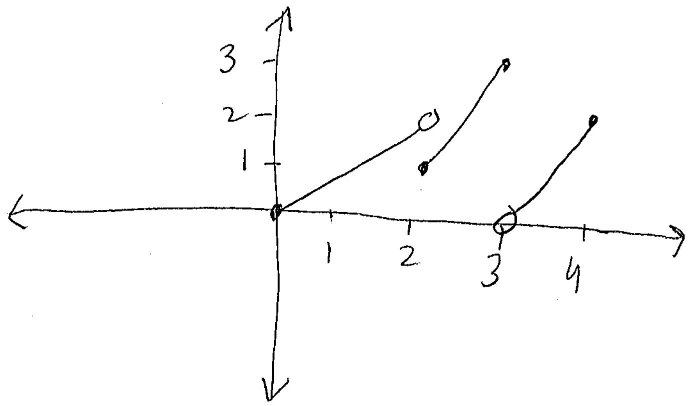
3 - Reading information from the graph of the function

Example 10: Consider the graph



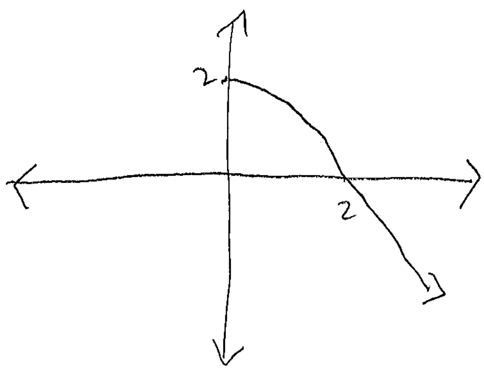
- $f(0)$
- $f(2)$
- $f(3)$
- Domain
- Range
- X-intercept!

Example 11: (Case-defined functions)



1. $f(0)$, $f(2)$, $f(3)$, $f(4)$
2. domain?
3. Range?
4. X-intercept

Exercise:

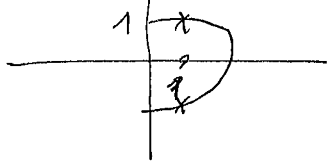


1. $f(0)$; $f(2)$
2. domain of f?
3. Range of f?
4. What is the X-intercept?

4- Vertical and horizontal line tests

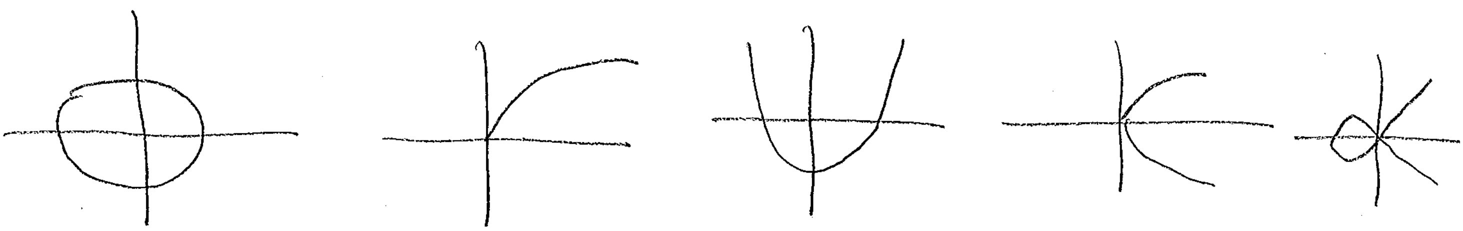
1- Vertical Line Test

Given a (curve) graph, how can we check if it is a graph of a function or <u>not</u>?



We apply the vertical line test (i.e., we draw vertical lines and we make sure it cuts the graph in only one point, otherwise we don't have a function).

<u>Example 12</u>: Test whether the following curves are graphs of functions or <u>not</u>?

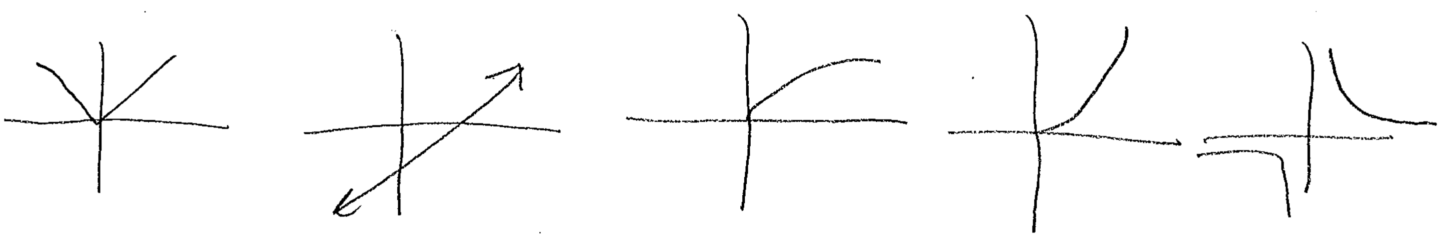


2- Horizontal Line Test

Given a (curve) graph, how can we check if the function has an inverse?

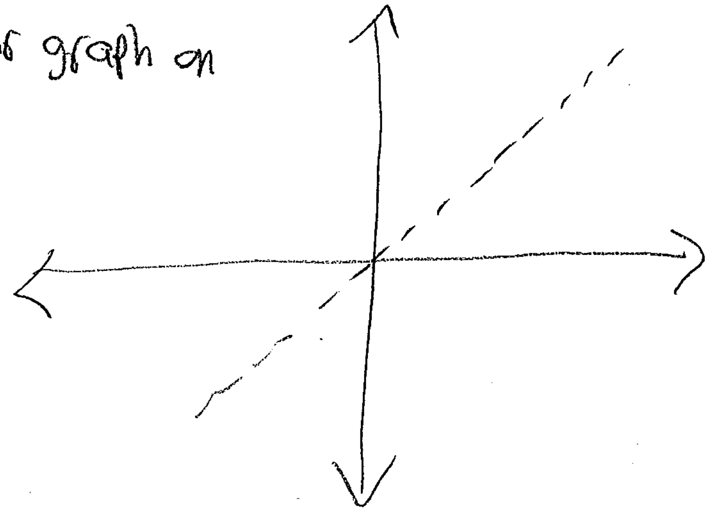
We apply the horizontal line test (i.e., we draw <u>horizontal</u> lines and we make sure it cuts the graph in only one point otherwise, it has <u>no</u> inverse).

<u>Example 13</u>: Test whether the following curves have an inverse or not?



5. Finding the graph of the inverse function from the graph of the original function

"Reflect your graph on $y = x$"



Example :