# § 3.1 - Lines

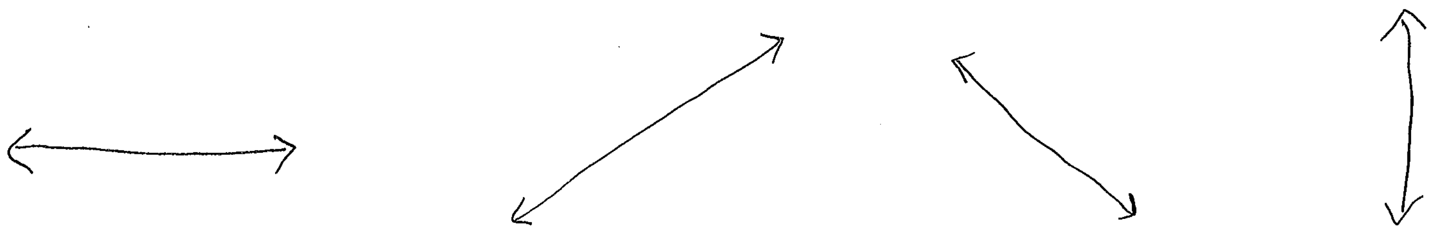
Recall: If we have a linear function $y = mx + b$, then by plotting two points on the graph of this function and connecting them, we get the graph of the function.

Goal: Given the graph of the line (i.e., we get two points $(x_1, y_1)$ and $(x_2, y_2)$). Find the equation of the line. (we will need to find a point and the slope of the line).

1. Slope
2. Equation of a line (multiple form).
3. parallel and perpendicular lines

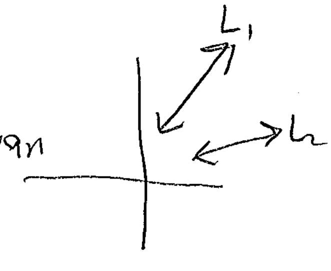
---

## 1- Slope

- The slope of a line is a number that measures how sloppy the line is (how hard to climb the stairs).
- 

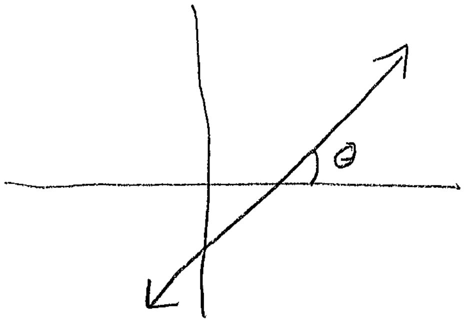
slope $= 0$      Slope $> 0$      Slope $< 0$      $\infty$ (undefined)

- consider the two lines $L_1$ & $L_2$ (both are of positive slope), but $L_1$ has slope greater than $L_2$



- Slope has a clear relation with the angle between the line and the x-axis.

if the slope rises, then $\theta$ rises!




---

## Finding the slope

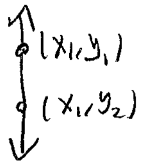
1. From the equation: (Solve the equation for $y$, i.e., let $y$ alone)

$$y = \underset{\substack{\uparrow \\ \text{slope.}}}{m} x + b$$

2. From the graph of the line, i.e., from two points on the line

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{vertical change}}{\text{horizontal change}}.$$

special case: The vertical line <u>has</u> <u>no</u> slope, why?!



Example 1:- Find the slope of the line that passes through

(1) (3, -1) and (6, 9)

(2) (7, 6) and (0, 1)

Solution:

(1) $(\underset{x_1}{3}, \underset{y_1}{-1})$ and $(\underset{x_2}{6}, \underset{y_2}{9})$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - (-1)}{6 - 3} = \frac{10}{3}$$

which means for every 3 steps to the right, we need to go 10 steps up.

(2) (7, -6) and (0, 1)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 7}{0 - (-6)} = \frac{-6}{6} = \frac{-1}{1}$$

which means for every one step to the right, we move one step down.

Exercise 1:

(1) (5, 2) and (4, -3)

(2) (1, 7) and (-9, 0)

(3) (5, 2) and (4, 2)

(4) (3, 1) and (3, 3)

## 2- Equation of the line

To get the equation of a line, you need to find ~~one~~

- one point $(x_1, y_1)$
- Slope $m$.

$$\boxed{y - y_1 = m(x - x_1)}$$ —— "point-slope form"

### other forms

- General linear form $ax + by + c = 0$, $a, b, c$ have no common factor.
- Slope-intercept form $y = mx + b$, $(0, b)$ is the y-intercept. $m$ is the slope.

Special case vertical line $\boxed{x = x_1}$

Example 2: Find a general linear equation ($ax + by + c = 0$) of the line with the following properties

(1) passes through $(1, -7)$ and has slope $-3$.

$$y - y_1 = m(x - x_1)$$

$$y - (-7) = -3(x - 1)$$

$$y + 7 = -3x + 3$$

$$y + 3x + 7 - 3 = 0$$

$$\boxed{y + 3x + 4 = 0}$$

(2) passes through $(-3, 4)$ and $(6, -4)$.

First we find the slope m.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - 4}{6 - (-3)} = \frac{-8}{9}$$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = \frac{-8}{9}(x + 3)$$

$$9(y - 36) = -8(x+3)$$

$$9y - 36 = -8x - 24$$

$$\boxed{9y + 8x - 12 = 0}$$

(3) has slope 4 and y-intercept -4

$$y = mx + b$$

$$y = 4x - 4$$

$$\boxed{y - 4x + 4 = 0}$$

(4) Is vertical and passes through $(-2, -7)$

$$x = x_1$$

$$\boxed{x = -2}$$

# 3. Parallel and Perpendicular Lines

- Two lines are <u>parallel</u> if

$$\boxed{m_1 = m_2}$$

- Two lines are <u>perpendicular</u> if

$$\boxed{m_1 m_2 = -1}$$

<u>Example 5</u>: Determine whether the given lines are parallel, perpendicular or neither?

(1) $y = -5x + 7$ and $y = -5x - 2$

$m_1 = -5$ and $m_2 = -5$, so the two lines are parallel.

(2) $x + 3y + 5 = 0$ <u>and</u> $y = +3x$

$3y = -x + 5$

$y = -\frac{1}{3}x + \frac{5}{3}$ $\qquad\qquad y = +3x$

$m_1 = -\frac{1}{3}$ $\qquad\qquad m_2 = 3 \rightarrow m_1 m_2 = \frac{-1}{3} \cdot 3 = -1$

So the two lines are <u>perpendicular</u>.

(3) $x - 2 = 3$ , $y = 2$

$x = 5$ $\qquad$ horizontal $\rightarrow$ they are <u>perpendicular</u>.

vertical

Exercise 5:

(1) $x+2y=0$ and $x+4y-7=0$

(2) $x=3$ and $x=-2$

(3) $y=4x+7$ and $4x-y+6=0$

---

Example 6: Find an equation of the line

(1) passes through $(2,1)$ and parallel to $y-4x+6=0$

we need to find the slope first, since both lines are parallel they have the same slope, so

$$m_1 = m_2 = 4.$$

$$y-y_1 = m_1(x-x_1)$$

$$y-1 = 4(x-1)$$

$$\boxed{y = 4x-3}$$

(2) Perpendicular to $2x+3y-2=0$ and passes through $(3,3)$

$$2x+3y-2=0 \quad\longrightarrow\quad m_2 = -\frac{2}{3}$$

$$3y = -2x+2 \qquad m_1 m_2 = -1 \rightarrow -\frac{2}{3} m_1 = -1$$

$$y = -\frac{2}{3}x+2 \qquad \boxed{m_1 = \frac{3}{2}}$$

The equation of the line

$$y - y_1 = m(x - x_1)$$

$$y - 3 = \frac{3}{2}(x - 3)$$

$$2y - 6 = 3(x - 3)$$

$$2y - 6 = 3x - 9$$

$$\boxed{2y - 3x + 3 = 0}$$

Exercise 8:

(1) parallel to $2x + 3y + 6 = 0$ and passes $(-7, -9)$

(2) perpendicular to $3y = \frac{-5}{2}x + 7$ and passes $(4, -1)$.

Old Exam Question

(a) Find the slope and the y-intercept of $3x + 6y - 12 = 0$.

(b) Find an equation of the line passing through $(2, -1)$ and parallel to $2y - 5 = 8x$.

(c) A computer manufacturer will produce 2500 units when the price is 82 BP and 1900 units when the price is 80 BP. Find the supply equation assuming it is linear.

(d) Find an equation of the line passing through $(2, 1)$ and perpendicular to the line $3y + x = 16$.