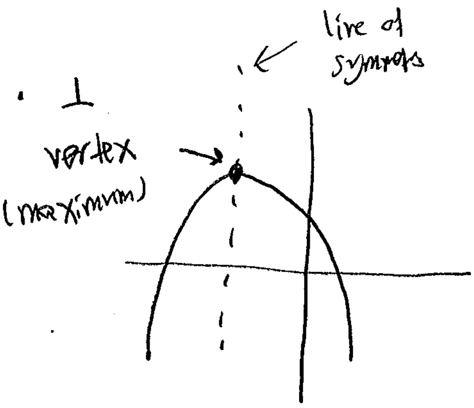
# § 3.3 – Quadratic Functions

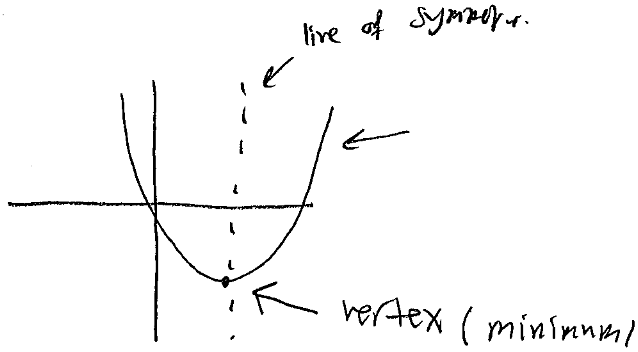
Recall: A quadratic function is

$$\boxed{f(x) = ax^2 + bx + c}, \quad a \neq 0$$

- The graph of a quadratic function is called parabola




$a < 0$
(open downward)




$a > 0$
(open upward)

- vertex is $\left(\frac{b}{-2a}, f\left(\frac{b}{-2a}\right)\right)$.

- y-intercept is $y = c$

- x-intercept is the solution of $ax^2+bx+c=0$ (use the formula Section 0.8).

Example 1: Graph $y = f(x) = -x^2 + 4x - 12$

Solution:

1- since $a = 1 > 0$, the parabola is upward.

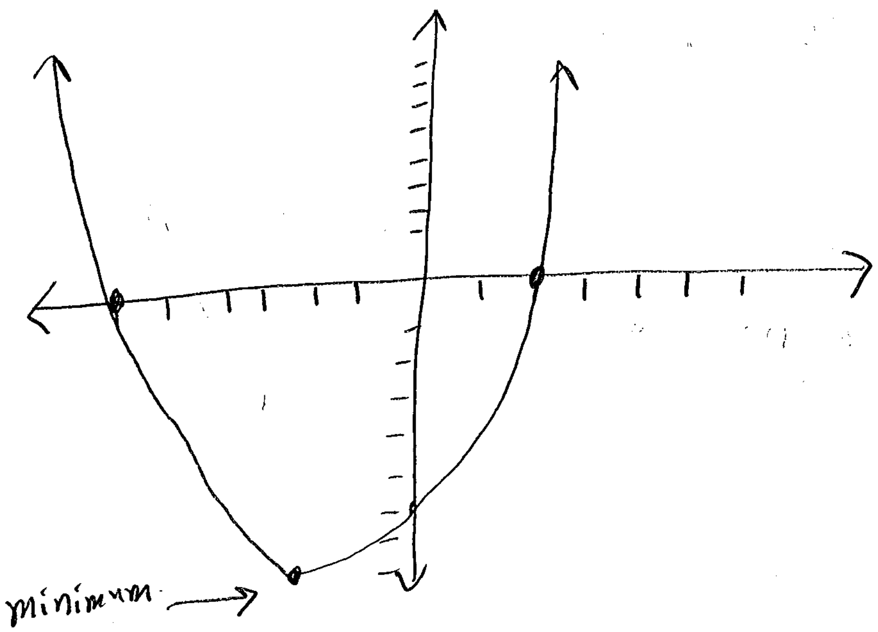
2- vertex $= \left(\frac{b}{-2a}, f\left(\frac{b}{-2a}\right)\right) = \left(\frac{4}{-2}, f\left(\frac{4}{-2}\right)\right) = (-2, f(-2)) = (-2, (-2)^2 + 4(-2) + 2)$
$= (-2, -16)$.

3- y-intercept $y = -12$, so the $(0, -12)$

4- x-intercept: we solve $x^2+4x-12=0$ to get

$x = -6$ or $x = 2$

$(-6, 0)$ or $(2, 0)$




Exercise 1: Graph $y = f(x) = -x^2+6x-5$.

Example 2: Graph $y = f(x) = x^2+4x+4$

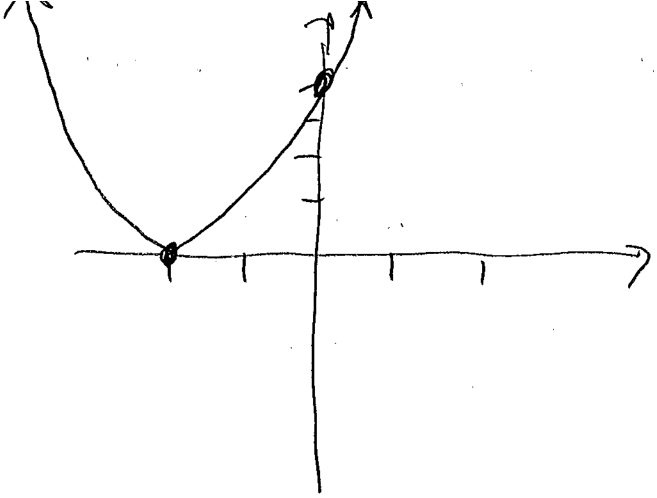
1. Since $a = 1 > 0$, the parabola is upward.

2. vertex: $\frac{b}{-2a} = \frac{4}{-2a} = -2$, so the vertex is $(-2, f(-2))$

$(-2, (-2)^2+4(-2)+4) = (-2, 0)$.

3. y-intercept: set $x=0 \to y = 4 \to (0, 4)$.

4- x-intercept: Solve $x^2+4x+4=0 \to x = -2 \to (-2, 0)$.



Exercise 2: Graph $y = f(x) = x^2 + x + 1$.

Example 3: The demand function is $p = f(q) = 4 - 2q$ where $p$ is the price & $q$ is # of units. Find the level of production that maximize the total revenue.

Solution:

$$\text{Total Revenue} = (\text{Price per unit})(\text{\# of units})$$
$$= (4 - 2q)(q)$$
$$= 4q - 2q^2$$

The maximum will be at the vertex, so we have

$$\text{maximum} = \frac{-b}{2a} = \frac{4}{-2(2)} = +1$$

So $q = 1$ & $p = 2$ is the maximum.

Exercise 3: (old Exam Question)
The demand function for a product is $p = 80 - 2q$. Find the quantity that maximize the revenue.

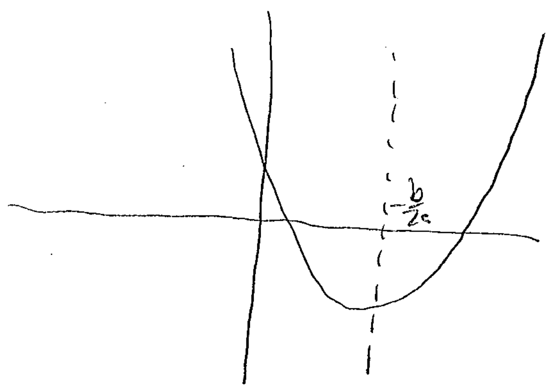
## Example 4: (Inverse of quadratic function)

(a) Does the quadratic function $f(x) = ax^2 + bx + c$ always have an inverse? If not, in which domain it has?

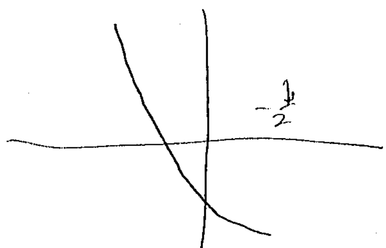
(b) Find the inverse of the quadratic function in a suitable domain.

Solution:

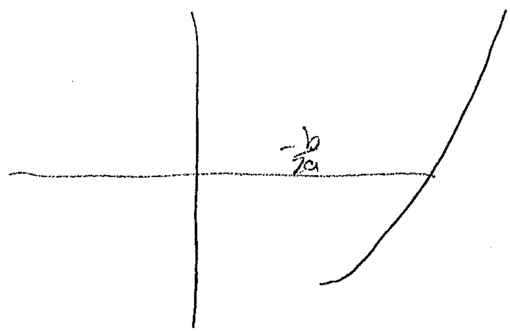
(a)




To apply the horizontal line test to the graph of $f(x)$, yield that it has no inverse, but if we restrict the domain




$\left(-\infty, -\frac{b}{2a}\right]$




$f(x) = ax^2 + bx + c$

Domain $= \left[-\frac{b}{2a}, \infty\right)$

(b) let the domain is $\left[-\frac{b}{2a}, \infty\right)$. Then $y = ax^2 + bx + c$

Step 1: Exchange $x$ & $y$ to get $x = ay^2 + by + c$

Step 2: Solve for $y$, so $ay^2 + by + c - x = 0$

$$y = \frac{-b \pm \sqrt{b^2 - 4a(c-x)}}{2a} \Rightarrow f^{-1}(x) = \frac{-b + \sqrt{b^2 - 4a(c-x)}}{2a}$$

which is not a quadratic function!