§ 3.4 - Systems of Linear Equations

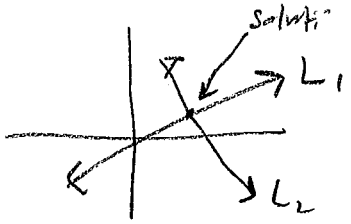
Definition:

A system of linear equations in two variables x & y is a list of linear equations.

$$\begin{aligned} a_1 x + b_1 y &= c_1 \\ a_2 x + b_2 y &= c_2 \\ &\vdots \\ a_n x + b_n y &= c_n \end{aligned}$$

Goal: Find value of x & y that are true for all the equation above at the same time.

one way: Graph the two equations



Example 1: (Elimination method)

$$x + 4y = 3 \quad \text{— (1)}$$
$$-3x + 2y = -5 \quad \text{— (2)}$$

we try to eliminate x first (you can do it with y).

$$3x + 12y = 9 \quad \text{— (1.1)}$$
$$-3x + 2y = -5 \quad \text{— (2.1)}$$
$$14y = 4 \longrightarrow \boxed{y = \frac{4}{14} = \frac{2}{7}}$$

$$x + 4\left(\frac{4}{14}\right) = 3$$
$$\boxed{x = \frac{13}{7}}$$

Solution set $= \{ (\underbrace{\frac{13}{7}}_{x} , \underbrace{\frac{2}{7}}_{y}) \}$

Note: check that your answer satisfies both equation (1) & (2)

Example 1:-

$$5x - 2y = 1 \quad -(1)$$

$$3x + 3y = 9 \quad -(2)$$

Solution:

$$15x - 6y = 3 \quad -(1)$$

$$-15x - 15y = -45$$

$$-21y = -42 \rightarrow \boxed{y = 2}$$

Substitute in (1), we get $5x - 2(2) = 1 \rightarrow \boxed{x = 1}$

Solution set $= \{ (1, 2) \}$

Exercise 1 :-

$$7x - 4y = -4$$

$$2x + 5y = -5$$

Example 3: (Substitution method)

Solve

$$2x - y = 1 \quad -(1)$$
$$-x + 2y = 7 \quad -(2)$$

Solution:

Using equation (1), isolate y in terms of x to get

$$y = 2x - 1 \quad -(3)$$

Now substitute (3) in (2) to get an equation in x only.

$$-x + 2(2x - 1) = 7$$

$$-x + 4x - 2 = 7 \longrightarrow 3x = 9 \longrightarrow \boxed{x = 3}$$

Substitute back in (3)

$$y = 2(3) - 1 = 5$$

Solution set $= \{(3, 5)\}$.

Exercise 2: Solve

$$-x + 2y = +7$$
$$5x + 3y = -9$$