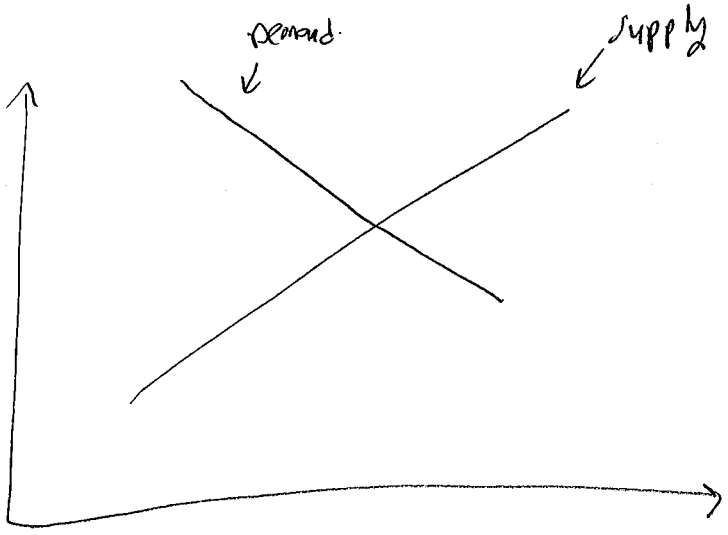
§3.6- Application of Systems of Equations



The equilibrium point is the point where the demand & supply meet

Example 1: Find the equilibrium point if the supply and demand equations are $p = \frac{q}{40} + 10$ and $p = \frac{8000}{q}$, respectively.

Solution:

We need to solve the system

$$p = \frac{q}{40} + 10 \quad \text{— (1)}$$

$$p = \frac{8000}{q} \quad \text{— (2)}$$

Substitute (2) in (1) to get $\frac{8000}{q} = \frac{q}{40} + 10$ (multiply by 40q)

$320000 = q^2 + 400q \rightarrow q^2 + 400q - 320000 = 0$

$q = 400$ or $q = -800$ (rejected/or disregarded).

$p = 20$

Example 2 & (old Exam Question)

Find the equilibrium point if supply and demand are

Supply $p = \sqrt{29+5q}$

Demand $p = 15 - q$

Moreover, find the revenue at the equilibrium point?

Solution:

We solve the system

$$p = \sqrt{29+5q} \quad -(1)$$

$$p = 15 - q \quad -(2)$$

Substitute (2) in (1) to get

$$15 - q = \sqrt{29+5q}$$

$$(15-q)^2 = \left(\sqrt{29+5q}\right)^2$$

$$225 - 30q + q^2 = 29 + 5q$$

$$q^2 - 35q + 196 = 0$$

$q = 28$ or $\boxed{q = 7}$ by the formula, Section 0.8

$p = -13$ (discarded)

$\boxed{p = 8}$

revenue = (price per unit)(# of units) $= 8(7) = \boxed{56}$

Example 3: (Break-Even points)

A manufacturer sells a product at 4 BD per unit. If the fixed cost is 2000 BD and variable cost is 2 BD per unit. Find the break-even point (i.e., the point where the total cost is equal to the total revenue (No profit)).

Solution: y_{TR} = Total revenue = (price per unit)(# of units) = $4q$

y_{TC} = Total cost = fixed cost + variable cost = $2000 + 2q$

In order to get the break-even point, we must have

$$y_{TC} = y_{TR}$$

$$4q = 2000 + 2q$$

$$2q = 2000 \rightarrow q = 1000$$

$$p = 4(1000) = 4000 \text{ BD}.$$

Exercise 2:

$$y_{TR} = 3\sqrt{q}, \quad y_{TC} = 2q + 500.$$

Exercise 3: (old Exam Question) Find the equilibrium point of

Demand: $25q - 2p + 320 = 0$

Supply: $45q + p - 505 = 0$

Exercise 4: (old final Exam Question)

For a certain product, the material cost is 9 BD per unit and the fixed cost is 5060 BD. If the price per unit is 6.5. Find the break-even quantity.