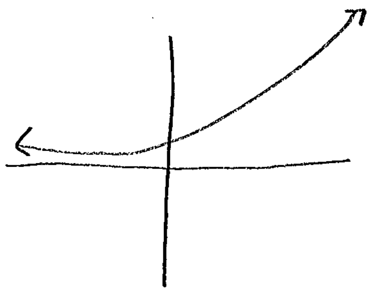
§ 4.2 - Logarithmic Function

1- The Logarithmic Function

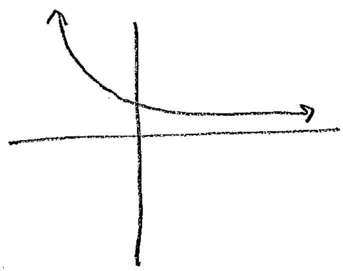
Recall: . The exponential function is

$$f(x) = a^x \quad , \quad a \neq 0, 1$$

. The general shape of $f(x) = a^x$ is



$a > 1$



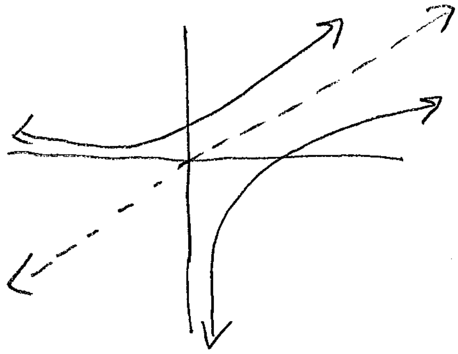
$0 < a < 1$

. Domain $= (-\infty, \infty)$

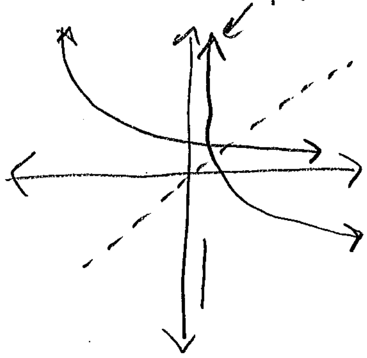
. Range $= (0, \infty)$

Question: Is $f(x)$ has an inverse?

Answer: Yes, using ~~horizontal~~ line test! The graph of $f^{-1}(x)$ is



$a > 1$



- $f^{-1}(x)$ is called the <u>logarithmic function base a</u> and is denoted by $f^{-1}(x) = \log_a x$.

<u>Note:</u> The Fundamental Equations

1- $f(f^{-1}(x)) = a^{f^{-1}(x)} = a^{\log_a x} = x$

2- $f^{-1}(f(x)) = \log_a a^x = x \longleftarrow$ very important!

2- Exponential and Logarithmic Form

$$\underbrace{\log_{a} x = \overset{\text{exponent}}{y}}_{\text{Logarithmic Form}} \quad \text{if and only if} \quad \underbrace{x = a^{\overset{\text{exponent}}{y}}}_{\text{exponential form}}$$

base, base

$\log_{\triangle} \boxed{2} = ③$ if and only if $\boxed{2} = \triangle^{③}$.

<u>Example 1:</u> Convert from logarithmic form to exponential form and vice versa.

1. $3^2 = 9 \iff 9 = 3^2 \iff \log_3 9 = 2$.

2. $\log_2 1024 = 10 \iff 1024 = 2^{10}$

3. $e^{-5} = y \iff \log_e y = -5$.

4- $8^{\frac{2}{3}} = 4 \iff \log_8 4 = \frac{2}{3}$.

5. $\log_2 \frac{1}{32} = -5 \iff \frac{1}{32} = 2^{-5}$.

6. $3^0 = 1 \iff \log_3 1 = 0$.

Exercise 1: Convert from exponentially given form to logarithmic given form.

(1) $\log_7 x = 5$ (2) $\log_2 \sqrt{2} = \frac{1}{2}$ (3) $9^3 = 729$

(4) $5^{\frac{1}{3}} = \sqrt[3]{5}$

"Jump to solving Equations".

Notation:

- If $a = 10$, then we simply write $\log_{10}$ to be $\log$ and it is called the common logarithm.

- If $a = e$, then we simply write $\log_e$ to be $\ln$ and it is called the natural logarithm.

Recall:

$$\log_a a^x = x.$$

Example 2: Evaluate the expression

(1) $\log_3 27 = \log_3 3^3 = 3 \log_3 3 = 3$. (4) $\log_6 \frac{1}{36} = \log_6 6^{-2} = -2$

(2) $\log 10{,}000 = \log 10^4 = \log_{10} 10^4 = 4$

(3) $\log_3 \sqrt[7]{3} = \log_3 3^{\frac{1}{7}} = \frac{1}{7}$

Exercise 2: Evaluate

(1) $\log 0.001$ (2) $\log_5 \sqrt[3]{25}$ (3) $\log_2 \sqrt[7]{4}$

(4) $\ln 11$ (5) $\log 60$.

Example 3: Solve for x

(1) $\log_3 x = 4$

Solution:

We convert it to exponential form to get

$$x = 3^4 = 81$$

Solution set $= \{81\}$

(2) $\log_x 4 = \frac{1}{2}$

$$4 = x^{\frac{1}{2}} \rightarrow 4 = x^2 \rightarrow x^2 - 4 = 0$$

So $x = 2$ or $x = -2$ (disregarded as the base cannot be negative)

Solution set $= \{2\}$

(3) $\log_4 x = -4 \rightarrow x = 4^{-4} = \frac{1}{256}$

Solution set $= \{\frac{1}{256}\}$.

Exercise 3: Solve for x

(1) $\log_5 x = 3$

(2) $\log_3 1 = 0$

(3) $\log_9 1 = 0$

(4) $\log_x (2x+8) = 2$