

## § 4.4 - Logarithmic and Exponential Equations

Recall:  $a^x = a^y \Leftrightarrow x = y$

$$\log_a x = \log_a y \Leftrightarrow x = y.$$

Example 1: Solve  $\log x - \log 5 = \log 7$

Solution:

$$\log \frac{x}{5} = \log 7$$

$$\frac{x}{5} = 7 \rightarrow x = 35$$

Solution set =  $\{35\}$

Example 2:  $(e^{(3x-2)})^3 = e^3$

Solution:

$$e^{3(3x-2)} = e^3$$

$$e^{9x-6} = e^3$$

$$9x-6 = 3$$

$$\boxed{x = 1}$$

Solution set =  $\{1\}$ .

Exercise (old exam)

Solve  $(\ln x + \ln 20) = 2x + 1$

$$e^{2x-1} = 27$$

Example 3: Solve  $(27)^{2x+1} = \frac{1}{3}$

Take  $\ln$  of both sides,

$$\ln(27)^{2x+1} = \ln \frac{1}{3}$$

$$(2x+1) \ln 27 = \ln \frac{1}{3}$$

$$(2x+1) \ln 3^3 = \ln 3^{-1}$$

$$(2x+1) = \frac{\ln 3^{-1}}{\ln 3^3}$$

$$(2x+1) = \frac{-\ln 3}{3 \ln 3}$$

$$2x+1 = -\frac{1}{3}$$

$$\boxed{x = -\frac{2}{3}}$$

$$\text{Solution set} = \left\{ -\frac{2}{3} \right\}$$

Exercise 1<sup>o</sup> solve  $16^{x+1} = 4^{2x}$

Example 3<sup>o</sup> solve  $10^{\frac{4}{x}} = 6$

Take  $\ln$  of both sides to get

$$\ln 10^{\frac{4}{x}} = \ln 6$$

$$\frac{4}{x} \ln 10 = \ln 6 \rightarrow \frac{4}{x} = \frac{\ln 6}{\ln 10} \rightarrow x = \frac{4 \ln 10}{\ln 6} = 4 \log_6 10.$$

Exercise 2<sup>o</sup> solve  $a^{(-4 + \frac{3}{2}x)} = 3$

Exercise 3<sup>o</sup> (old exam question)

$$3e^{2x-3} - 4 = 2$$

Example 4: Solve  $7^{3x+2} = 8$

Solution:

Take  $\ln$  of both sides,

$$\ln 7^{3x+2} = \ln 8$$

$$(3x+2) \ln 7 = \ln 8$$

$$(3x+2) = \frac{\ln 8}{\ln 7} \rightarrow 3x = \frac{\ln 8}{\ln 7} - 2$$

$$x = \frac{\frac{\ln 8}{\ln 7} - 2}{3}$$

$$\text{Solution set} = \left\{ \frac{\frac{\ln 8}{\ln 7} - 2}{3} \right\}$$

Example 5:  $\log_4 (x-2) = 1$

$$(x-2) = 4^1 \rightarrow x = 6, \text{ solution set} = \{6\}$$

Example 3: Solve  $\log_2 (x-1) = 6$

Example 6: Solve  $\log_2 x + \log_2 (x-1) = 1$

Solution:

$$\log_2 x + \log_2 (x-1) = 1$$

$$\log_2 x(x-1) = 1$$

$$x(x-1) = 2^1 \rightarrow x^2 - x = 2 \rightarrow x^2 - x - 2 = 0$$

Solution set =  $\{2\}$   $\boxed{x=2}$  or  $x = -1$  (disregarded as we cannot have negative inside the log).  $\boxed{3}$

Exercise 4:  $\log(x-3) + \log(x-5) = 1$

Example 7: Solve  $\log(x+2) - \log x = 2$

Solution:  $\log \frac{x+2}{x} = 2$

$$\frac{x+2}{x} = 10^2$$

$$\frac{x+2}{x} = 100$$

$$x+2 = 100x$$

$$2 = 100x - x$$

$$2 = 99x$$

$$\boxed{\frac{2}{99} = x}$$

$$\text{Solution set} = \left\{ \frac{2}{99} \right\}$$

Exercise 5: Solve  $\log(x+5) = \log(3x+2) + 1$

Exercise 2 (old exam question) solve  $\log_2 x + \log_2(x+2) = 3$

Exercise 2 (old <sup>final</sup> exam question) solve  $\log_2 + \log(4-x) = 2 \log x$