§ 5.1 - Compound Interest

Recall (Section 4.1)

The compound interest formula is given by

$$A = P\left(1+\frac{r}{m}\right)^{n.m}$$

where,

P = original (invested) money (Principal).

A = ~~number~~ of years that we are invested.

m = number of period ~~their~~ year to receive interest.

r = annual interest rate which is called nominal rate or annual percentage rate (A.P.R).

A = accumulated amount.

$I = A - P$ = compounded interest.

(See the examples from Section 4.2)

Example 1: How long it takes for 600 BD to amount to 800 BD at an annual rate of 4% compounded quarterly?

$P = 600$, $A = 800$, $n = ?$, $m = 4$, $r = 4\% = 0.04$.

$$A = P\left(1+\frac{r}{m}\right)^{n.m}$$

$$800 = 600\left(1+\frac{0.04}{4}\right)^{4n} \rightarrow 800 = 600(1.01)^{4m}$$

$$\frac{4}{3} = (1.01)^{4m} \rightarrow \ln\frac{4}{3} = 4m \cdot \ln 1.01 \rightarrow 4m = \frac{\ln\frac{4}{3}}{\ln 1.01}$$

$n \approx 7.22$ years.

Exercise 1: Suppose 400 BD amounted to 580 BD in an Saving account with interest rate 3% Compounded semi-annually. Find the number of years?

Example 2: (Find the rate)

Suppose 100 BD amounted to 160 BD in six years. If the interest was compounded quartely, find the nomial rate, that was earned by the money.

Solution:

$P = 100, \quad A = 160, \quad r = ?, \quad n = 6, \quad m = 4.$

$$A = P\left(1 + \frac{r}{m}\right)^{n \cdot m}$$

$$160 = 100\left(1 + \frac{r}{4}\right)^{6 \cdot 4}$$

$$\frac{160}{100} = \left(1 + \frac{r}{4}\right)^{24}$$

$$1.6 = \left(1 + \frac{r}{4}\right)^{24}$$

$$\sqrt[24]{1.6} = 1 + \frac{r}{4}$$

$$1.01977 = 1 + \frac{r}{4}$$

$$0.01977 = \frac{r}{4}$$

$$0.079 = r$$

$$\boxed{7.9\% = r}$$

Exercise 2: At what nominal rate of interest, compounded yearly, will 1 BD doubled in 10 years?

Example 3: The inflation rate in Bahrain for October 2015 is 2.75 In. How many years we will have to pay 2 BD for something that we buy now at 1.6 BD?

$P = 1.6 \qquad A = 2 \text{ BD} \quad , r = 2.75\% = 0.0275 , \; n = ? \; , m = 1$

$$A = P\left(1 + \frac{r}{m}\right)^{n \cdot m}$$

$$2 = 1.6\left(1 + \frac{0.0275}{1}\right)^{n \cdot 1}$$

$$\frac{2}{1.6} = (1.0275)^n \longrightarrow n = \frac{\ln \frac{2}{1.6}}{\ln 1.0275} = 8.22 \text{ years}$$

Exercise 3: Same as the previous example with inflation rate of 7% (as in 2008) and for 1 BD to double.

$P \longrightarrow P r_e + P$ (year)

$P \longrightarrow P\left(1+\frac{r}{m}\right)^m = A$

Effective Rate:

$$r_e = \left(1 + \frac{r}{m}\right)^m - 1$$

This is the rate of simple interest in one year.

$$A_{simple} = A_{compounded}$$

$$P + P r_e = P\left(1 + \frac{r}{m}\right)^m \longrightarrow P r_e = P\left(1 + \frac{r}{m}\right)^m - P$$

$$P r_e = P\left[\left(1 + \frac{r}{m}\right)^m - 1\right] \rightarrow r_e = \left(1 + \frac{r}{m}\right)^m - 1$$

Example 4: What is the effective rate to a nomial rate of 4% compunded

(a) yearly: $r_e = \left(1+\frac{0.04}{1}\right)^1 - 1 = 1.04 - 1 = 4\%$

(b) Semi-annually: $r_e = \left(1+\frac{0.04}{2}\right)^2 - 1 = 0.0404 = 4.04\%$

(c) quartely: $r_e = \left(1+\frac{0.04}{4}\right)^4 - 1 = 0.0406 = 4.06\%$

(d) monthly: $r_e = \left(1+\frac{0.04}{12}\right)^{12} - 1 = 0.0407 = 4.07\%$

(e) daily: $r_e = \left(1+\frac{0.04}{365}\right)^{365} - 1 = 0.0408 = 4.080\%$

Exercise 4: Same as the previous example with rate 7%.

Example 5: An investor has a choice of investing a sum of money at 8% compounded annually or at 7.8% compounded semi-annually. Which is the better option?

Solution: we need to compare the effective rate (the real rate in one year).

option 1: $r_e = \left(1+\frac{r}{m}\right)^m - 1 = \left(1+\frac{0.08}{1}\right)^1 - 1 = 0.08 = 8\%$

option 2: $r_e = \left(1+\frac{7.8}{2}\right)^2 - 1 = \quad = 0.079 = 7.9\%$

So the first option is better.

Exercise: Compare 5% daily or 5.1% quartely