

§ 5.1 - Compound Interest

Recall (Section 4.1)

The compound interest formula is given by

$$A = P \left(1 + \frac{r}{m} \right)^{n \cdot m}$$

where,

P = original (invested) money (Principal).

A = ~~number~~ number of years that we are invested.

m = number of period ~~that~~ _{interest} year to receive interest.

r = annual rate which is called nominal rate or annual percentage rate (A.P.R).

A = accumulated amount.

$$I = A - P = \text{interest}$$

(See the examples from section 4.2)

Example 1: How long it takes for 600 BD to amount to 800 BD at an annual rate of 4% compounded quarterly?

$$P = 600, \quad A = 800, \quad n = ?, \quad m = 4, \quad r = 4\% = 0.04.$$

$$A = P \left(1 + \frac{r}{m} \right)^{n \cdot m}$$

$$800 = 600 \left(1 + \frac{0.04}{4} \right)^{4n} \rightarrow 800 = 600 (1.01)^{4n}$$

$$\frac{4}{3} = (1.01)^{4n} \rightarrow \ln \frac{4}{3} = 4n \cdot \ln 1.01 \rightarrow 4n = \frac{\ln \frac{4}{3}}{\ln 1.01}$$

$n \approx 7.22$ years.

Exercise 1: Suppose 400 BD amounted to 580 BD in an saving account with interest rate 3% compounded semi-annually. Find the number of years?

Example 2: (Find the rate)

Suppose 100 BD amounted to 160 BD in six years. If the interest was compounded quarterly; find the nominal rate, that was earned by the money.

Solution:

$$P = 100, \quad A = 160, \quad r = ?, \quad n = 6, \quad m = 4.$$

$$A = P \left(1 + \frac{r}{m} \right)^{n \cdot m}$$

$$160 = 100 \left(1 + \frac{r}{4} \right)^{6 \cdot 4}$$

$$\frac{160}{100} = \left(1 + \frac{r}{4} \right)^{24}$$

$$1.6 = \left(1 + \frac{r}{4} \right)^{24}$$

$$\sqrt[24]{1.6} = 1 + \frac{r}{4}$$

$$1.01977 = 1 + \frac{r}{4}$$

$$0.01977 = \frac{r}{4}$$

$$0.079 = r$$

$$\boxed{7.9\% = r}$$

Exercise 2: At what nominal rate of interest, compounded yearly, will 1 Bp doubled in 10 years?

Example 3: The inflation rate in Bahrain for October 2015 is 2.75%. How many years we will have to pay 2 Bp for something that we buy now at 1.6 Bp?

$$P = 1.6 \quad A = 2 \text{ Bp} \quad , \quad r = 2.75\% = 0.0275, \quad n = ? \quad , \quad m = 1$$

$$A = P \left(1 + \frac{r}{m}\right)^{n \cdot m}$$

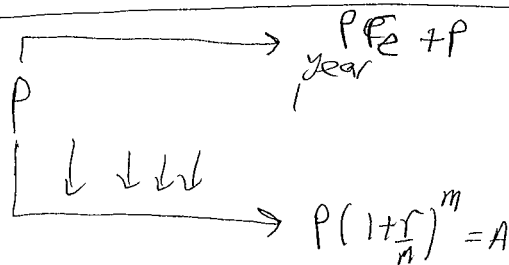
$$2 = 1.6 \left(1 + \frac{0.0275}{1}\right)^{n \cdot 1}$$

$$\frac{2}{1.6} = (1.0275)^n \rightarrow n = \frac{\ln \frac{2}{1.6}}{\ln 1.0275} = 8.22 \text{ years}$$

Exercise 3: Same as the previous example with inflation rate of 7% (as in 2008) and for 1 Bp to double.

Effective Rate

$$r_e = \left(1 + \frac{r}{m}\right)^m - 1$$



This is the rate of simple interest in one year.

$$A_{\text{simple}} = A_{\text{compounded}}$$

$$P + P r_e = P \left(1 + \frac{r}{m}\right)^m \rightarrow$$

$$P r_e = P \left(1 + \frac{r}{m}\right)^m - P$$

$$P r_e = P \left[\left(1 + \frac{r}{m}\right)^m - 1\right] \rightarrow r_e = \frac{\left(1 + \frac{r}{m}\right)^m - 1}{1}$$

Example 4: What is the effective rate to a nominal rate of 4% compounded

(a) yearly: $r_e = \left(1 + \frac{0.04}{1}\right)^1 - 1 = 1.04 - 1 = 4\%$

(b) Semi-annually: $r_e = \left(1 + \frac{0.04}{2}\right)^2 - 1 = 0.0404 = 4.04\%$

(c) Quarterly: $r_e = \left(1 + \frac{0.04}{4}\right)^4 - 1 = 0.0406 = 4.06\%$

(d) monthly: $r_e = \left(1 + \frac{0.04}{12}\right)^{12} - 1 = 0.0407 = 4.07\%$

(e) daily: $r_e = \left(1 + \frac{0.04}{365}\right)^{365} - 1 = 0.0408 = 4.08\%$

Exercise 4: Same as the previous example with rate 7%.

Example 5: An investor has a choice of investing a sum of money at 8% compounded annually or at 7.8% compounded semi-annually. Which is the better option?

Solution: we need to compare the effective rate (the real rate in one year).

option 1: $r_e = \left(1 + \frac{r}{m}\right)^m - 1 = \left(1 + \frac{0.08}{1}\right)^1 - 1 = 0.08 = 8\%$

option 2: $r_e = \left(1 + \frac{7.8}{2}\right)^2 - 1 = 0.079 = 7.9\%$

So the first option is better.

Exercise: Compare 5% daily or 5.7% quarterly