§5.3 - Interest Compounded Continuously

Recall: The compound interest formula is given by

$$A = P\left(1 + \frac{r}{m}\right)^{n \cdot m}$$

Where m is the number of times an interest is paid. Suppose interest r is paid frequent (i.e., m is a large number (i.e., $m \to \infty$)) then how much money should we earn?

Answer:

Recall, $e = \lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{x}$, so we have

$$A = P\left(1 + \frac{r}{m}\right)^{n \cdot m} = P\left(1 + \frac{1}{\frac{m}{r}}\right)^{n \cdot m}, \text{ set } X = \frac{m}{r} \to m = Xr$$

$$A = P\left(1 + \frac{1}{X}\right)^{n \cdot Xr} = P\left[\left(1 + \frac{1}{X}\right)^{X}\right]^{nr}, \text{ now let } X \to \infty$$

$$\boxed{A = Pe^{rn}} \quad \text{--- compound amount under continuous interest}$$

Note: Think of it as having an interest every second!!

Effective rate: $r_e = e^{r} - 1$

Present value: $P = Ae^{-rn}$.

Example 1: If 1000 BD is deposited in a saving account that earns interest at an annual rate of 5.5% compound continuously What is the value of the account at the end of 3 years?

Solution:

$P = 1000$, $A = ?$, $r = 5.5\% = 0.055$, $n = 3$.

$$A = Pe^{rn} = 1000\, e^{(0.055)(3)} = 1179.39 \text{ BD}.$$

Exercise 1: Find the compound amount and compound interest if 400 BD is invested for 5 years in an account with interest compound continuously

(a) $5\frac{1}{4}\%$ (b) 10%

Exercise 2: Find the ~~effective~~ rate that corresponds to the given annual rate compounded continuously.

(a) 3% (b) 7% (c) 2% (d) 10%

Example 2: What is the annual ~~rate~~ rater corresponding to ~~effectin~~ compounded continuously rate of 5%?

$$r_e = e^r - 1$$

$$0.05 = e^r - 1 \rightarrow e^r = 1.050 \rightsquigarrow r = \ln 1.05$$

$$r \sim 4.88\%$$