

§6.2 - Matrix Addition and Scalar Multiplication

Let $A = (A_{ij})$, $B = (B_{ij})$, then
of the same size

1. Matrix Addition
 $A + B = (A_{ij} + B_{ij})$ "adding coordinatewise"

2. Scalar Multiplication
 $c \cdot A = (cA_{ij})$ "multiply everything by c"

Example 1:

Let $A = \begin{pmatrix} -2 & 1 \\ 2 & -3 \end{pmatrix}$, $B = \begin{pmatrix} -5 & -5 \\ 3 & -3 \end{pmatrix}$, and $C = \begin{pmatrix} -7 & -1 \\ 3 & -3 \end{pmatrix}$, $D = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

Find:

(i) $A + B = \begin{pmatrix} -2-5 & 1-5 \\ 2+3 & -3-3 \end{pmatrix} = \begin{pmatrix} -7 & -4 \\ 5 & -6 \end{pmatrix}$

(ii) $2B = \begin{pmatrix} 2(-5) & 2(-5) \\ 2(3) & 2(-3) \end{pmatrix} = \begin{pmatrix} -10 & -10 \\ 6 & -6 \end{pmatrix}$

(iii) $2A - 3B = \begin{pmatrix} -4 & 2 \\ 4 & -6 \end{pmatrix} - \begin{pmatrix} -15 & -15 \\ 9 & -9 \end{pmatrix} = \begin{pmatrix} 11 & 17 \\ -5 & 3 \end{pmatrix}$

(iv) $3A + C^T = \begin{pmatrix} -6 & 3 \\ 6 & -9 \end{pmatrix} + \begin{pmatrix} -1 & 3 \\ -3 & -3 \end{pmatrix} = \begin{pmatrix} -7 & 6 \\ 3 & -12 \end{pmatrix}$

(v) $(2A - B)^T = \left(\begin{pmatrix} -4 & 2 \\ 4 & -6 \end{pmatrix} - \begin{pmatrix} -5 & -5 \\ 3 & -3 \end{pmatrix} \right)^T = \begin{pmatrix} 1 & 7 \\ 1 & 9 \end{pmatrix}^T = \begin{pmatrix} 1 & 1 \\ 7 & 9 \end{pmatrix}$

Exercise 1: (old exam question)

Let $A = \begin{pmatrix} -3 & 1 & 5 \\ 2 & 1 & 0 \end{pmatrix}$ and $C = \begin{pmatrix} 2 & 5 \\ 6 & 3 \\ 0 & -4 \end{pmatrix}$, Find $3A - 2C^T$
 $2A + \text{?}$ $\sqrt{1}$

Example 2: Solve the matrix equation

$$\begin{pmatrix} 4 \\ 6 \\ 8 \end{pmatrix} + 2 \begin{pmatrix} x \\ y \\ 2z \end{pmatrix} = \begin{pmatrix} -10 \\ -24 \\ 20 \end{pmatrix}$$

Solution:

$$\begin{pmatrix} 4 \\ 6 \\ 8 \end{pmatrix} + \begin{pmatrix} 2x \\ 2y \\ 4z \end{pmatrix} = \begin{pmatrix} -10 \\ -24 \\ 20 \end{pmatrix}$$

$$\begin{pmatrix} 4+2x \\ 6+2y \\ 8+4z \end{pmatrix} = \begin{pmatrix} -10 \\ -24 \\ 20 \end{pmatrix} \rightarrow \begin{array}{l} 4+2x=10 \\ 6+2y=-24 \\ 8+4z=20 \end{array} \rightarrow \begin{array}{l} x=3 \\ y=-15 \\ z=5 \end{array}$$

Solution Set = $\{ (3, -15, 5) \}$