§6.3 : Matrix Multiplication

Goal: To define multiplication between two matrices A & B.

When it is possible?

$$A_{m\times n} \cdot B_{n\times k} = AB_{m\times k}.$$

must be the same

Example 1: (1) $A = \begin{pmatrix} 1 & 3 & 2 \end{pmatrix}_{1\times3}$, $B = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}_{3\times1}$

$$A\cdot B = \begin{pmatrix} 1 & 3 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} = (1+3+6) = (10).$$

(2) $A = \begin{pmatrix} 1 & 2 \\ -3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ 5 & 6 \end{pmatrix}$

- $AB = \begin{pmatrix} 1 & 2 \\ -3 & 4 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 5 & 6 \end{pmatrix} = \begin{pmatrix} 11 & 12 \\ 17 & 24 \end{pmatrix}$

- $BA = \begin{pmatrix} 1 & 0 \\ 5 & 6 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ -3 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ -13 & 34 \end{pmatrix}.$

(3) $A = \begin{pmatrix} -1 & 1 \\ 0 & 4 \\ 2 & 1 \end{pmatrix}_{3\times2}$, $B = \begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix}_{2\times3}$

$$AB = \begin{pmatrix} -1 & 1 \\ 0 & 4 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 2 & 6 \\ 12 & 16 \\ 5 & 0 \end{pmatrix}.$$

(4) $A = \begin{pmatrix} -1 & -1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$

$\cdot\ AA^T = \begin{pmatrix} -1 & -1 & 0 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ -1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}.$

$\cdot\ A^2 = A \cdot A = \begin{pmatrix} -1 & -1 & 0 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} -1 & -1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$ Not possible.

(5) $A = \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 2 & -3 \\ -1 & \frac{3}{2} \end{pmatrix}$

$AB = \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & -3 \\ -1 & \frac{3}{2} \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \rightsquigarrow AB = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ while $A \neq 0$, $B \neq 0$.

So the integrality property Does Not hold in matrices!

"If $a \cdot b = 0 \to a = 0$ or $b = 0$"

<u>Exercise:</u> (old Exam Question)

(i) $B = \begin{pmatrix} 1 & 5 & 2 \\ 0 & -2 & -4 \\ 3 & 0 & 5 \end{pmatrix}$, $C = \begin{pmatrix} 2 & 5 \\ 6 & 3 \\ 0 & -4 \end{pmatrix}$. Find BC, $C^T B$

(ii) $A = \begin{pmatrix} 2 & 5 \\ -1 & 4 \\ 0 & -3 \end{pmatrix}$, $B = \begin{pmatrix} 6 & 10 & 7 \\ -2 & 3 & 1 \end{pmatrix}$, $C = \begin{pmatrix} 8 & -4 \\ -1 & 2 \end{pmatrix}$

(i) C^2 (ii) $BA - 5I$.

Example 2: Represent each system of linear equations as product of matrices

(i) $3x + y = 6$

$2x - 9y = 5$

$$\downarrow$$

$$\begin{array}{cc} x & y \\ \left(\begin{array}{cc} 3 & 1 \\ 2 & -9 \end{array}\right) \end{array} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 5 \end{pmatrix}$$

(ii) $3x + y + z = 2$

$x - y + z = 4$

$5x - y + 2z = 12$

$$\downarrow$$

$$\begin{pmatrix} 3 & 1 & 1 \\ 1 & -1 & 1 \\ 5 & -1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 12 \end{pmatrix}$$