Section 0.8
Quadratic Equations

Dr. Abdulla Eid

College of Science

MATHS 103: Mathematics for Business I

Recall: A *linear equation* is an equation of the form $ax + b = 0$.

Definition

A **quadratic equation** in one variable x is an equation of the form

$$ax^2 + bx + c = 0, \qquad a \neq 0, b, c \text{ are real numbers}$$

To solve quadratic equation, we have two ways

- Factoring.
- Quadratic formula (powerful tool, but takes some time)

1. Solving by factoring

Example

Solve the equation $x^2 + x - 12 = 0$.

Solution:

$$x^2 + x - 12 = 0$$

$$(x + 4)(x - 3) = 0$$

$$x + 4 = 0 \text{ or } x - 3 = 0$$

$$x = -4 \text{ or } x = 3$$

Solution Set$= \{-4, 3\}$

Exercise

Solve the equation $t^2 + 3t + 2 = 0$.

Example

Solve the equation $6w^2 - 5w = 0$.

Solution: We take w as a common factor to get

$$w(6w - 5) = 0$$

$$w = 0 \text{ or } (6w - 5) = 0$$

$$w = 0 \text{ or } w = \frac{5}{6}$$

Solution Set$=\{0, \frac{5}{6}\}$

Open bracket

Example

Solve the equation $(3x-4)(x+1)=-2$.

Solution: We multiple the two bracket to make the equation in the standard form.

$$\begin{aligned} 3x^2+3x-4x-4 &= -2 \\ 3x^2-x-2 &= 0 \\ (3x+2)(x-1) &= 0 \\ 3x+2=0 \text{ or } x-1 &= 0 \\ x = \frac{-2}{3} \text{ or } x &= 1 \end{aligned}$$

Solution Set$=\{\frac{-2}{3}, 1\}$

Taking common factor

Exercise

Solve the equation $4x - 4x^3 = 0$.

Exercise

Solve the equation $x(x+2)^2(x+5)+x(x+2)^3=0$.

2. Solving by quadratic formula

Definition

The **two** solution for the quadratic equation $ax^2 + bx + c = 0$ are given

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example

Solve $x^2 + x - 24 = 0$

Solution: Here $a = 1$, $b = 1$, and $c = -24$. We substitute into the quadratic formula to get

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(-24)}}{2(1)}$$

$$x = \frac{-1 \pm \sqrt{97}}{2} \Rightarrow \text{ Solution Set } = \{\frac{-1 - \sqrt{97}}{2}, \frac{-1 + \sqrt{97}}{2}\}$$

Exercise

Solve the equation $9y^2 + 6\sqrt{2}y + 2 = 0$.

Note: Sometimes we get only *one* solution to the quadratic equation.

Exercise

Solve the equation $z^2 + z + 1 = 0$.

Note: Sometimes we get *no* solution to the quadratic equation.

Example

Solve $\frac{1}{x^6} + \frac{9}{x^3} + 8 = 0$.

Solution: We multiply both sides by x^6 to get

$$1 + 9x^3 + 8x^6 = 0.$$

which is the same as the equation

$$1 + 9(x^3) + 8(x^3)^2 = 0.$$

in which it has two solutions

$$x^3 = -1 \text{ or } x^3 = \frac{-1}{8}$$

$$\sqrt[3]{-1} = \sqrt[3]{x^3} \text{ or } \sqrt[3]{\frac{-1}{8}} = \sqrt[3]{x^3}$$

$$-1 = x \text{ or } \frac{-1}{2} = x$$

Solution Set $= \{-1, \frac{-1}{2}\}$.