Section 1.2
Linear Inequality

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MATHS 103: Mathematics for Business I

Comparison Operators

Let a and b be two real numbers of the number line.

- $a = b$.
- $a < b$ which is read as a " is less than " b.
- $a > b$ which is read as a " is greater than " b.
- $a \leq b$ which is read as a " is less than or equal to " b.
- $a \geq b$ which is read as a " is greater than or equal to " b.

Intervals

Let $a < b$.
Recall:

- $(a, b] : \quad a < x \leq b.$
- $[a, b) : \quad a \leq x < b.$
- $(a, \infty) : \quad a < x.$
- $(-\infty, a) : \quad x < a.$
- $(-\infty, \infty) : \quad -\infty < x < \infty.$

Exercise

Describe each of the following interval in the number line and inequality symbols:

1. $[2, 5]$.
2. $(-5, \infty)$.
3. $(-\infty, 0]$.
4. $[a, \infty)$.
5. $(0, 1)$.
6. $[0, 1]$.

Definition

An *inequality* is a mathematical statement that uses inequality symbols ($<$, $>$, $\leq$, $\geq$) to connect two expressions.

Goal: To solve inequalities that contain a variable. The answer will be written in the form of an interval, inequality symbols, and will be described in the number line.

Definition

A **linear inequality** in the variable x is an inequality that can be written in the form

$$ax + b < 0, \qquad \text{or } >, \leq, \geq$$

Rules of inequality:

- If $A + c < B$, then $A < B - c$. (As we did for the equations).
- If $cA < cB$, then $A < B$ if $c > 0$. (As we did for the equations).
- If $cA < cB$, then $A > B$ if $c < 0$. (Exception of what we did for the equations).

Example

Solve $2(x-3) < 4$

Solution: We solve linear inequality in the same way we solve linear equations (see Section 0.7) except if we multiply or divide by a negative number, then we need to reverse the inequality sign.

$$
\begin{aligned}
2(x-3) &< 4 \\
2x-6 &< 4 \\
2x &< 4+6 \\
2x &< 10 \\
\frac{2x}{2} &< \frac{10}{2} \\
x &< 5
\end{aligned}
$$

The Solution

1- Set notation

$$\text{Solution Set} = \{x \mid \underbrace{x < 5}_{\text{your answer}} \}$$

2- Number Line notation

3- Interval notation

$$(-\infty, 5)$$

You can check your answer by picking any value in the solution set and substitute it in the original inequality.Take $x = 0$. Then

$$2(0-3)\boxed{\phantom{<}}4 \rightarrow -6\boxed{\phantom{<}}4$$

which we can fill it with $<$.

Exercise

Solve $3 < 2y + 3$

Example

Solve $4 - 2x \leq 6$

Solution:

$$\begin{aligned} 4 - 2x &\leq 6 \\ -2x &\leq 6 - 4 \\ -2x &\leq 2 \\ \frac{-2x}{-2} &\geq \frac{2}{-2} \\ x &\geq -1 \end{aligned}$$

The Solution

1- Set notation

$$\text{Solution Set} = \{x \mid -1 \leq x\}$$

2- Number Line notation

3- Interval notation

$$[-1, \infty)$$

Exercise

Solve $-3 \geq 8(2 - x)$

Example

Solve $\frac{3}{2}(x-2)+1 > -2(x-4)$

Solution:

$$\begin{aligned} \frac{3}{2}(x-2)+1 &> -2(x-4) \\ 2\left(\frac{3}{2}(x-2)\right)+2(1) &> 2(-2(x-4)) \\ 3(x-2)+2 &> -4(x-4) \\ 3x-6+2 &> -4x+16 \\ 3x+4x &> 16+4 \\ 7x &> 20 \\ \frac{7x}{7} &> \frac{20}{7} \\ x &> \frac{20}{7} \end{aligned}$$

The Solution

1- Set notation

$$\text{Solution Set} = \{x \mid x > \frac{20}{7}\}$$

2- Number Line notation

3- Interval notation

$$(\frac{20}{7}, \infty)$$

Special Cases

Example

Solve $2(x-2) > 2x+6$

Solution:

$$\begin{aligned} 2(x-2) &> 2x+6 \\ 2x-4 &> 2x+6 \\ 2x-2x &> 6+4 \\ 0 &> 10 \end{aligned}$$

Which is not true at all. Hence in the case that x disappears and the last statement we got is incorrect we say that we don't have a solution!.

The Solution

1- Set notation

$$\text{Solution Set} = \{\} = \emptyset \quad (\text{"The empty set})$$

2- Number Line notation

3- Interval notation

$$\emptyset$$

Special Cases

Example

Solve $2x + 6 < 2(x + 5)$

Solution:

$$2x + 6 < 2(x + 5)$$
$$2x + 6 < 2x + 10$$
$$2x - 2x < 10 - 6$$
$$0 < 4$$

Which is always true. Hence in the case that x disappears and the last statement we got is correct we say that all real numbers are solution. In short,

$$-\infty < x < \infty$$

The Solution

1- Set notation

$$\text{Solution Set} = \{x \mid -\infty < x < \infty\}$$

2- Number Line notation

3- Interval notation

$$(-\infty, \infty)$$