Section 1.4
Absolute Value

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MATHS 103: Mathematics for Business I

Absolute Value

Definition

The **absolute value** of any number x is the *distance* between x and the zero. We denote it by $|x|$.

Example

- $|2|$ = distance between 2 and 0 = 2.
- $|-3|$ = distance between -3 and 0 = 3.
- $|0|$ = distance between 0 and 0 = 0.
- $|-2|$ = distance between -2 and 0 = 2.

Note: The absolute value $|x|$ is always **non–negative**, i.e., $|x| \geq 0$.

Properties of Absolute Values

1. $|ab| = |a| \cdot |b|$.
2. $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$.
3. $|a - b| = |b - a|$.
4. $|a + b| \leq |a| + |b|$.
5. $-|a| \leq a \leq |a|$.

Rules

For equations (or inequalities) that involve absolute value we need to get rid of the absolute value which can be done only using the following three rules:

1. Rule 1: $|X| = a \rightarrow X = a$ or $X = -a$.
2. Rule 2: $|X| < a \rightarrow -a < X < a$.
3. Rule 3: $|X| > a \rightarrow X > a$ or $X < -a$.

Example

Solve $|x-3|=2$

Solution: We solve the absolute value using rule 1 to get rid of the absolute value.

$$|x-3|=2$$
$$x-3=2 \text{ or } x-3=-2$$
$$x=5 \text{ or } x=1$$

Solution Set $=\{5,1\}$.

Exercise

Solve $|7 - 3x| = 5$

Example

Solve $|x-4| = -3$

Solution: Caution: The absolute value can never be negative, so in this example, we have to stop and we say there are no solution!
Solution Set $= \{\} = \emptyset$.

Exercise

(Old Exam Question) Solve $|7x + 2| = 16$.

Example

Solve $|x-2|<4$

Solution:

$$
\begin{aligned}
|x-2| &< 4 \\
-4 < x-2 &< 4 \\
-4+2 < x &< 4+2 \\
-2 < x &< 6
\end{aligned}
$$

The Solution

1- Set notation

$$\text{Solution Set} = \{x \mid -2 < x < 6\}$$

2- Number Line notation

3- Interval notation

$$(-2, 6)$$

Exercise

Solve $|3 - 2x| \leq 5$

Example

Solve $|x+5| \leq -2$

Solution: Caution: The absolute value can never be negative or less than a negative, so in this example, we have to stop and we say there are no solution!
Solution Set $= \{\} = \emptyset$.

Exercise

(Old Final Exam Question) Solve $|5 - 6x| \leq 1$

Exercise

(Old Final Exam Question) Solve $|2x - 7| \leq 9$

Example

Solve $|x+5| \geq 7$

Solution:

$$|x+5| \geq 7$$
$$x+5 \geq 7 \text{ or } x+5 \leq -7$$
$$x \geq 2 \text{ or } x \leq -12$$

The Solution

1- Set notation

$$\text{Solution Set} = \{x \mid x \geq 2 \text{ or } x \geq -12\}$$

2- Number Line notation

3- Interval notation

$$(-\infty, -12] \cup [2, \infty)$$

where $\cup$ means union of two intervals.

Exercise

Solve $|3x - 4| > 1$

Example

Solve $\left|\frac{3x-8}{2}\right| \geq 4$

Solution:

$$\left|\frac{3x-8}{2}\right| \geq 4$$

$$\frac{3x-8}{2} \geq 4 \text{ or } \frac{3x-8}{2} \leq -4$$

$$3x-8 \geq 8 \text{ or } 3x-8 \leq -8$$

$$3x \geq 16 \text{ or } 3x \leq 0$$

$$x \geq \frac{16}{3} \text{ or } x \leq 0$$

The Solution

1- Set notation

$$\text{Solution Set} = \{x \,|\, x \geq \frac{16}{3} \text{ or } x \geq 0\}$$

2- Number Line notation

3- Interval notation

$$(-\infty, 0] \cup [\frac{16}{3}, \infty)$$

Exercise

(Old Exam Question) Solve $|x+8|+3<2$

Exercise

(Old Exam Question) Solve $|10x - 9| \geq 11$.