Section 2.1
Functions

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MATHS 103: Mathematics for Business I

1. Definition of a function.
2. Finding the domain of a function.
3. Finding function values.
4. Application of functions.

1. Definition of a function

A **function** from a set X to a set Y is an *assignment* (*rule*) that tells how one element x in X is related to only one element y in Y.

Notation:

- $f : X \to Y$.
- $y = f(x)$. "f of x".
- x is called the input (independent variable) and y is called the output (dependent variable).
- The set X is called the domain and Y is called the co–domain. While the set of all outputs is called the range.

Think about the function as a vending machine!

Question: How to describe a function mathematically?

Answer: By using algebraic formula!

Example

Consider the function

$$\begin{aligned} f : (-\infty, \infty) &\to (-\infty, \infty) \\ x &\mapsto 3x + 1 \end{aligned}$$

or simply by $f(x) = 3x + 1$

- f(1)=3(1)+1=4.
- f(0)=3(0)+1=1.
- f(-2)=3(-2)+1=-5.
- f(-7)=3(-7)+1=-20.

- Domain $= (-\infty, \infty)$.
- Co–domain$=(-\infty, \infty)$.
- Range$=(-\infty, \infty)$.

Example

$$f : (-\infty, \infty) \to (-\infty, \infty)$$
$$x \mapsto x^2$$

or simply by $f(x) = x^2$

- f(1)=$(1)^2$=1.
- f(0)=$(0)^2$=0.
- f(-1)=$(-1)^2$=1.
- f(-2)=$(-2)^2$=4.
- f(14)=$(-4)^2$=16.
- f(4)=$(4)^2$=16.

- Domain $= (-\infty, \infty)$.
- Co–domain$=(-\infty, \infty)$.
- Range$=[0, \infty)$.

Example

$$f : (-\infty, \infty) \to (-\infty, \infty)$$
$$x \mapsto \frac{1}{x}$$

or simply by $f(x) = \frac{1}{x}$

- $f(1)=\frac{1}{1}=1$.
- $f(-1)=\frac{1}{-1}=-1$.
- $f(2)=\frac{1}{2}=\frac{1}{2}$.
- $f(-4)=\frac{1}{-4}=\frac{-1}{4}$.
- $f(100)=\frac{1}{100}=\frac{1}{100}$.
- $f(0)=\frac{1}{0}=$undefine (Problem, so we have to exclude it from the domain!)

- Domain $= \{x \mid x \neq 0\}$.
- Co–domain$=(-\infty, \infty)$.
- Range$=\{y \mid, y \neq 0\}$.

2. Finding the domain of functions

Recall:

The *domain* of f is the set of all x such that $f(x)$ defined (makes sense). I.e., no problems like having a zero in the denominator or negative inside the square root, etc). so

$$\text{Domain of } f = \{x \mid f(x) \text{ defined } \}$$

Example

(Zero denominator) Find the domain of $f(x) = \frac{3}{x-1}$.

Solution: Here we would have problems (undefined values) only if the denominator is equal to zero, so we need to find when the denominator is equal to zero and we exclude them from the domain.

$$x - 1 = 0 \to x = 1$$

So the domain of f is the set of all values except $x = 1$

The domain

1- Set notation

$$\text{Domain} = \{x \mid \underbrace{x \neq 1}_{\text{your answer}} \}$$

2- Number Line notation

3- Interval notation

$$(-\infty, 1) \cup (1, \infty)$$

Exercise

Find the domain of $f(x) = \frac{2x+1}{3x+8}$.

Example

(Zero denominator) Find the domain of $f(x) = \frac{x^2-1}{3x^2-5x-2}$.

Solution: Similarly to the previous example, we would have problems (undefined values) only if the denominator is equal to zero, so we need to find when the denominator is equal to zero and we exclude them from the domain.

$$3x^2 - 5x - 2 = 0$$

$$x = 2 \quad \text{or} \quad x = \frac{-1}{3} \qquad (\text{Section 0.8 using the formula})$$

So the domain of f is the set of all values except $x = 2$ and $x = -\frac{1}{3}$

The domain

1- Set notation

$$\text{Domain} = \{x \mid x \neq 2 \text{ and } x \neq -\frac{1}{3}\}$$

2- Number Line notation

3- Interval notation

$$(-\infty, -\frac{1}{3}) \cup (-\frac{1}{3}, 2) \cup (2, \infty)$$

Exercise

Find the domain of $f(x) = \frac{21}{x^2+3}$.

Example

(Negative inside the root) Find the domain of $f(x) = \sqrt{2x - 4}$.

Solution: Here we would have problems (undefined values) only if there is a negative inside the square root, so we need to find all values that make $2x - 4$ is greater than or equal to zero, so we need to solve the inequality

$$2x - 4 \geq 0 \rightarrow x \geq 2$$

So the domain of f is the set of all values x such that $x \geq 2$

The domain

1- Set notation

$$\text{Domain} = \{x \mid x \geq 2\}$$

2- Number Line notation

3- Interval notation

$$[2, \infty)$$

Example

(Negative inside the root and zero in the denominator) Find the domain of $f(x) = \frac{3}{\sqrt{x-4}}$.

Solution: Here we would have two problems (undefined values) only if there is a negative inside the square root or zero in the denominator, so we need to find all values that make $x-4$ is is equal to zero and we exclude them. Then we find all the values that make $x-4$ non–negative, so we need to solve the first

$$\text{denominator} = 0 \qquad \text{and} \qquad \text{inside} \geq 0$$

$$x - 4 = 0 \text{ and } x - 4 \geq 0$$

So the domain of f is the set of all values x such that $x \geq 4$ and $x \neq 4$

The domain

1- Set notation

$$\text{Domain} = \{x \mid x \geq 4 \text{ and } x \neq 4\}$$

2- Number Line notation

3- Interval notation

$$(4, \infty)$$

Exercise

Find the domain of $f(x) = \frac{3x^2+1}{\sqrt{3x+6}}$.

Exercise

Find the domain of $f(x) = \frac{\sqrt{x+5}}{2x-4}$.

Exercise

(Old Exam Question) Find the domain of the following functions:

- $f(x) = 2x + 5$.
- $g(x) = \frac{4}{x^2-4}$.
- $h(x) = \sqrt{3x+1}$.

Example

(Negative inside two roots) Find the domain of $f(x) = \sqrt{2 - \sqrt{x}}$.

Solution: Here we would have problems (undefined values) only if there is a negative inside the square roots, so we need to find all values that make whatever inside the square root to be greater than or equal to zero, so we need to solve two inequalities at the same time

$$x \geq 0 \qquad \text{and} \qquad 2 - \sqrt{x} \geq 0$$

$$x \geq 0 \qquad \text{and} \qquad 4 \geq x$$

So the domain of f is the set of all values x such that $x \geq 0$ and $4 \geq x$.

The domain

1- Set notation

$$\text{Domain} = \{x \mid x \geq 0 \text{ and } 4 \geq x\}$$

2- Number Line notation

3- Interval notation

$$[0, 4]$$

Finding Function Values

Recall

$$(a \pm b)^2 = a^2 \pm 2ab + b^2$$

Example

Let $g(x) = x^2 - 2$. Find

- f(2)=$(2)^2 - 2$=2. (we replace each x with 2).
- f(u)=$(u)^2 - 2$=$u^2 - 2$.
- f(u^2)=$(u^2)^2 - 2$=$u^4 - 2$.
- f($u+1$)=$(u+1)^2 - 2$=$u^2 + 2u + 1 - 2 = u^2 + 2u - 1$.

Exercise

Let $f(x) = \frac{x-5}{x^2+3}$. Find

- $f(5)$.
- $f(2x)$.
- $f(x+h)$.
- $f(-7)$.

Example

Let $f(x) = x^2 + 2x$. Find $\frac{f(x+h)-f(x)}{h}$.

Solution:

$$
\begin{aligned}
\frac{f(x+h)-f(x)}{h} &= \frac{(x+h)^2 + 2(x+h) - (x^2+2x)}{h} \\
&= \frac{x^2 + 2xh + h^2 + 2x + 2h - x^2 - 2x}{h} \\
&= \frac{2xh + h^2 + 2h}{h} \\
&= \frac{h(2x+h+2)}{h} \\
&= 2x + h + 2
\end{aligned}
$$

Exercise

Let $f(x) = 2x^2 - x + 1$. Find $\frac{f(x)-f(2)}{x-2}$.

4. Application of Functions

Example

(Demand Function) The demand function is expressed as the following for certain item

$$p = \frac{120}{q},$$

where q is the number of units and p is the price for unit. If the price is 6 BD per unit, how many units we have?

Solution:

$$p = \frac{120}{q}$$

$$6 = \frac{120}{q}$$

$$6q = 120$$

$$q = \frac{120}{6} = 20$$

Example

A company has capital of 7000 BD and weekly income of 320 BD and weekly expenses of 210 BD. Find the value V of the company in terms of t which is the number of weeks.

Solution:

$$V(t) = 7000 + (320 - 210)t$$

$$V(t) = 7000 + 110t$$