Section 2.3
Combination of Functions

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MATHS 103: Mathematics for Business I

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Goal: To create *new* functions from *old* ones. This will define **operations** between functions that are similar to the operations between numbers.

1- Arithmetic Operations

Example

Let $f(x) = 2x^2$ and $g(x) = x - 1$. Now a new function

$$h(x) = f(x) + g(x) = 2x^2 + x - 1$$

which is just adding the output of the function f and g. We denote h simply by $f + g$. Therefore,

$$(f + g)(x) = f(x) + g(x)$$

Note: $f + g$ is a function, while $(f + g)(x)$ is its output on input x.

Arithmetic Operations on Functions

Let f, g are two functions on the same domain and c is a constant.

1. Sum:

$$(f+g)(x) = f(x) + g(x)$$

2. Difference:

$$(f-g)(x) = f(x) - g(x)$$

3. Product:

$$(f \cdot g)(x) = f(x) \cdot g(x)$$

4. quotient:

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \qquad (g(x) \neq 0))$$

5. Scalar Product:

$$(c \cdot f)(x) = c \cdot f(x) \qquad \text{special case of (3)}$$

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Example

If $f(x) = x^2 - 1$ and $g(x) = x^2 + 2x$. Find

1. $(f+g)(x) = f(x) + g(x) = (x^2-1) + (x^2+2x) = 2x^2 + 2x - 1.$
2. $(f-g)(3) = f(3) - g(3) = (3^2-1) - (3^2+2(3)) = -7.$
3. $(f \cdot g)(x) = f(x) \cdot g(x) = (x^2-1) \cdot (x^2+2x) = x^4 + 2x^3 - x^2 - 2x.$
4. $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^2-1}{x^2+2x}.$

Exercise

Find the domain of $\frac{f}{g}$ from the previous example

2- Composition of Functions

This is a very important way to create a new function from old ones and will be used in Chapter 4 and in MATHS 104 next semester.

Suppose we have $f : X \to Y$ and $g : Y \to Z$. We define $g \circ f : X \to Z$ by

$$(g \circ f)(x) = g(f(x))$$

The function f is called the **inner** function and the function g is called the **outer** function and $\circ$ is called "**composite**" operation.

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Example

If $f(x) = \sqrt{x}$ and $g(x) = x - 1$. Find

1. $(f \circ g)(x) = f(g(x)) = \sqrt{g(x)} = \sqrt{x-1}$.
2. $(g \circ f)(x) = g(f(x)) = f(x) - 1 = \sqrt{x} - 1$.

Exercise

Find the domain of $f \circ g$ and $g \circ f$ and show that they are different function.

Note: In general,

$$f \circ g \neq g \circ f$$

(Look at the previous example).

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Exercise

Let $f(x) = x + 3$ and $g(x) = x^2$. Find $(f \circ g)(x)$ and $(g \circ f)(x)$.

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Example

If $f(x) = \frac{1}{x^2+1}$ and $g(x) = \sqrt{x+2}$. Find

1. $(f \circ g)(x) = f(g(x)) = \frac{1}{(g(x))^2+1} = \frac{1}{(\sqrt{x+2})^2+1} = \frac{1}{x+2+1} = \frac{1}{x+3}$.
2. $(g \circ f)(x) = g(f(x)) = \sqrt{f(x)+2} = \sqrt{\frac{1}{x^2+1}+2}$.

Exercise

(Homework) Find the domain of $f \circ g$ and $g \circ f$.

Example

(Old Exam Question) Let $f(x) = 2x + 5$, $g(x) = \frac{4}{x^2-9}$, $h(x) = \sqrt{3x+1}$. Find the following

1. $(f+g)(1) = f(1) + g(1) = 7 + \frac{-1}{2} = \frac{13}{2}$.
2. $(g \circ h)(x) = g(h(x)) = \frac{4}{(h(x))^2-9} = \frac{4}{(\sqrt{3x+1})^2-9} = \frac{4}{3x+1-9} = \frac{4}{3x-8}$.
3. $(h \circ f)(x) = h(f(x)) = \sqrt{3(f(x))+1} = \sqrt{3(2x+5)+1} = \sqrt{6x+16}$.
4. $(f \circ g)(1) = f(g(1)) = f(\frac{-1}{2}) = 2(\frac{-1}{2}) + 5 = 4$.

Exercise

Find the domain of $(h \circ f)(x)$ and $(g \circ h)(x)$ from the example above.

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Finding the inner and outer functions

This will be useful later in MATHS 104.

Example

Find f and g such that $h(x) = g(f(x))$.

1. $h(x) = \sqrt{x^2 - 2}$.
2. $h(x) = \frac{3}{x^2+x+1}$.
3. $h(x) = \sqrt[4]{\frac{x^2-1}{x+3}}$.

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Exercise

Find f and g such that $h(x) = g(f(x))$.

1. $h(x) = (2x-1)^3$.
2. $h(x) = \frac{2-(3x-5)}{(3x-5)^2+2}$..

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