Section 2.4
Inverse Functions

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MATHS 103: Mathematics for Business I

1. Definition of inverse function.
2. Finding the inverse function.

1- Definition of inverse function

Recall:

- If a is a number, then $a - a = 0 = -a + a$. $-a$ is an **inverse** of a with respect to the addition $+$.
- If a is a non-zero number, then $a\frac{1}{a} = 1 = \frac{1}{a}a$. $\frac{1}{a}$ is an **inverse** of a with respect to the multiplication $\cdot$.
- If f is a function, we want to find an "**inverse**" g to f *with respect to the composite* $\circ$, i.e., we want to find g (which is called the inverse) such that

$$(f \circ g)(x) = x \text{ and } (g \circ f)(x) = x$$

  usually, we denote it by f^{-1}.

If f is a "nice" function, we want to find an "inverse" g

Note: Not every function has an inverse! (we will see the horizontal line test later).

Finding the inverse function

Step 0: Write $y = f(x)$.
Step 1: Exchange x and y in step 0.
Step 2: Solve the literal equation in step 1 for y (see Section 0.7).

Example

(Old Exam Question) Find the inverse of $g(x) = 5x - 3$.

Solution: Step 0: Write $y = g(x)$.

$$y = 5x - 3$$

Step 1: Exchange x and y in step 0.

$$x = 5y - 3$$

Step 2: Solve the literal equation in step 1 for y

$$x = 5y - 3$$
$$x + 3 = 5y$$

Continue...

Step 2: Solve the literal equation in step 1 for y

$$
\begin{aligned}
x &= 5y - 3 \\
x + 3 &= 5y \\
\frac{x+3}{5} &= y
\end{aligned}
$$

Hence we have

$$g^{-1}(x) = \frac{x+3}{5}$$

To check you answer, we have to check that $g(g^{-1}(x)) = x$ and $g^{-1}(g(x)) = x$.

1.
$$\begin{aligned} g(g^{-1}(x)) &= 5(g^{-1}(x)) - 3 \\ &= \left(\frac{x+3}{5}\right) - 3 \\ &= (x+3) - 3 \\ &= x \end{aligned}$$

2.
$$\begin{aligned} g^{-1}(g(x)) &= \frac{g(x)+3}{5} \\ &= \frac{(5x-3)+3}{5} \\ &= \frac{5x}{5} \\ &= x \end{aligned}$$

Exercise

Find the inverse of $F(x) = (4x - 5)^2$.

Exercise

Find the inverse of $y = \frac{3}{2}x + \frac{7}{5}$.