Section 2.5
Graphs in Rectangular Coordinates

Dr. Abdulla Eid

College of Science

MATHS 103: Mathematics for Business I

1. Rectangular Coordinate System.
2. Graphing functions, intercepts, domain and range.
3. Reading information from the graph of the function.
4. Vertical and Horizontal line tests.
5. Finding the graph of the inverse function from the graph of the original function.

1. Rectangular Coordinate System

Motivational Example: Consider the number line

How do we describe the position of the point in red?

We take another copy of the number line and we put it perpendicular to the old one so that their origins coincide

- P is described by an *ordered* pair

$$(\underbrace{1}_{x\text{--coordinate}}, \underbrace{2}_{y\text{--coordinate}}) \qquad \text{The order is important}$$

- The special point is $(0, 0)$ called the **origin**.
- To locate any point, the x–coordinate tells you how many steps horizontally (either left or right) and the y–coordinate tells you how many steps to go vertically (up or down).
- There are four quadrants in the rectangular plane.

Exercise

Label each point and give which quadrant
$(-2, -5)$, $(3, -1)$, $(-\frac{1}{3}, \frac{1}{2})$, $(1, 0)$, $(-4, 5)$, $(3, 0)$, $(0, -6)$.

2- Graphs, intercepts, domain, and range

Example

Graph (sketch) the function $y = x^2 - 1$.

We substitute values of x to find the values of y and we fill the table

x	-2	-1	0	1	2	3
y						

Note:

- In ideal world, we will need to plot infinitely many points to get a perfect graph, but this is not possible, so our concern is only on the "general shape" of the function by joining only several points by a smooth curve whenever possible.
- In MATHS104, we will be able to graph more complicated functions in an easier way! (using calculus).

Intercepts

- The x–intercept is the point where the graph of the function intersects with the x–axis.
- The y–intercept is the point where the graph of the function intersects with the y–axis.

To find:

- x–intercept Set $y = 0$ and find the missing x.
- y–intercept Set $x = 0$ and find the missing y.

Example

Find the x–intercept and the y–intercept of the graph $y = 3x - 6$ and sketch its graph. Find also the domain and the range.

Solution:

1- x–intercept: Set $y = 0$ and fing the missing x. We get,

$$\begin{aligned} 0 &= 3x - 6 \\ 6 &= 3x \\ \frac{6}{3} &= x \\ 2 &= x \end{aligned}$$

So the x–intercept is the point

$$(\underbrace{2}_{x}, \underbrace{0}_{y})$$

Continue...

Example

Find the x–intercept and the y–intercept of the graph $y = 3x - 6$ and sketch its graph. Find also the domain and the range.

2- y–intercept: Set $x = 0$. Then

$$y = 3(0) - 6 = -6$$

So the x–intercept is the point

$$(\underbrace{0}_{x}, \underbrace{-6}_{y})$$

3- Sketch: plot some extra point to get the general shape and join them:

x	0	2	-2	-1	1	3
y	-6	0				

- Domain $= (-\infty, \infty)$.
- Co–domain$=(-\infty, \infty)$. Range$=(-\infty, \infty)$.

Exercise

Find the x–intercept and the y–intercept of the graph $y = 3 - 2x$ and sketch its graph. Find also the domain and the range.

Example

Find the x–intercept and the y–intercept of the graph $y = 4x^2 - 16$ and sketch its graph. Find also the domain and the range.

Solution:

1- x–intercept: Set $y = 0$. Then,

$$0 = 4x^2 - 16$$

$$x = 2 \quad \text{or} \quad x = -2 \quad \text{by the formula in Section 0.8}$$

So the x–intercept are the points

$$(2, 0) \qquad \text{and} \qquad (-2, 0)$$

Continue...

Example

Find the x–intercept and the y–intercept of the graph $y = 4x^2 - 16$ and sketch its graph. Find also the domain and the range.

2- y–intercept: Set $x = 0$. Then

$$y = 4(0)^2 - 16 = -16.$$

So the x–intercept is the point

$$(0, -16)$$

3- Sketch: plot some extra point to get the general shape and join them:

x	0	2	-2	-1	1	3
y	-16	0	0			

- Domain $= (-\infty, \infty)$.
- Co–domain$=(-\infty, \infty)$. Range$=[-16, \infty)$.

Example

Find the x–intercept and the y–intercept of the graph $y = \frac{-1}{x}$ and sketch its graph. Find also the domain and the range.

Solution:

1- x–intercept: Set $y = 0$. Then,

$$0 = \frac{-1}{x}$$
$$0 \cdot x = \frac{-1}{x} \cdot x$$
$$0 = -1$$

Which has *no* x and it is always false. So there is no x–intercept.

Continue...

Example

Find the x–intercept and the y–intercept of the graph $y = \frac{-1}{x}$ and sketch its graph. Find also the domain and the range.

2- y–intercept: Set $x = 0$. Then

$$y = \frac{-1}{0}$$

which is undefined and hence there is no solution and thus no y–intercept.

3- Sketch: plot some extra point to get the general shape and join them:

x	-3	-2	-1	$\frac{-1}{2}$	$\frac{-1}{3}$	$\frac{-1}{4}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	1	2	3
y												

- Domain $= \{x \mid x \neq 0\}$.
- Co–domain$=(-\infty, \infty)$.
- Range$=\{y \mid y \neq 0\}$.

Example

Find the x–intercept and the y–intercept of the graph $y = \sqrt{x}$ and sketch its graph. Find also the domain and the range.

Solution: 1- x–intercept: Set $y = 0$. Then,

$$0 = \sqrt{x}$$
$$0 = x$$

So the x–intercept is the point

$$(0, 0)$$

Continue...

Example

Find the x–intercept and the y–intercept of the graph $y = \sqrt{x}$ and sketch its graph. Find also the domain and the range.

2- y–intercept: Set $x = 0$. Then

$$y = \sqrt{0} = 0$$

So the x–intercept is the point

$$(0, 0)$$

3- Sketch: plot some extra point to get the general shape and join them:

x	0	1	2	3	4	9
y	0					

- Domain $= [0, \infty)$.
- Co–domain$=(-\infty, \infty)$.
- Range$=[0, \infty)$.

Exercise

(Old Exam Question) Find the x–intercept and the y–intercept of the graph $y = \sqrt{3x+1}$ and sketch its graph. Find also the domain and the range.

Exercise

Find the x–intercept and the y–intercept of the graph $y = |x|$ and sketch its graph. Find also the domain and the range.

Example

Find the x–intercept and the y–intercept of the graph $y = |3x + 2|$ and sketch its graph. Find also the domain and the range.

Solution: 1- x–intercept: Set $y = 0$. Then,

$$0 = |3x + 2|$$
$$0 = 3x + 2 \qquad \text{Section 1.4}$$
$$x = \frac{-2}{3}$$

So the x–intercept is the point

$$(\frac{-2}{3}, 0)$$

Continue...

Example

Find the x–intercept and the y–intercept of the graph $y = |3x + 2|$ and sketch its graph. Find also the domain and the range.

2- y–intercept: Set $x = 0$. Then

$$y = |3(0) + 2| = 2.$$

So the x–intercept is the point

$$(0, 2)$$

3- Sketch: plot some extra point to get the general shape and join them:

x	0	$\frac{-2}{3}$	-2	-1	1	3
y	2	0				

- Domain $= (-\infty, \infty)$.
- Co–domain$=(-\infty, \infty)$.
- Range$=[0, \infty)$.

Example

Find the x–intercept and the y–intercept of the graph $y = \sqrt{x^2 - 4}$ and sketch its graph. Find also the domain and the range.

Solution:

1- x–intercept: Set $y = 0$. Then,

$$\begin{aligned} 0 &= \sqrt{x^2 - 4} \\ (0)^2 &= (\sqrt{x^2 - 4})^2 \\ 0 &= x^2 - 4 \\ x = 2 \quad &\text{or} \quad x = -2 \qquad \text{by the formula in Section 0.8} \end{aligned}$$

So the x–intercept are the points

$$(2, 0) \qquad \text{and} \qquad (-2, 0)$$

Continue...

Example

Find the x–intercept and the y–intercept of the graph $y = \sqrt{x^2 - 4}$ and sketch its graph. Find also the domain and the range.

2- y–intercept: Set $x = 0$. Then

$$y = \sqrt{0^2 - 4} = \sqrt{-4}.$$

So no solution and so there is no y–intercept.
3- Sketch: plot some extra point to get the general shape and join them:

x	2	-2	-4	-3	-1	0	1	3	4
y	0	0							

- Domain $= (-\infty, -2) \cup (2, \infty)$.
- Co–domain$=(-\infty, \infty)$.
- Range$=[0, \infty)$.

Summary...

- So far, we are given the function $y = f(x)$ and we are asked to sketch the graph of the function.
- Now, we will be given the graph of the function (without the algerbaic expression of the function) and we will be asked to find some important information about the function.

3 - Reading Information from the graph of the function

Example

For the following graph, find

1. $f(0)$, $f(2)$, $f(3)$.
2. Domain of f.
3. Range of f.
4. What is the x–intercept.

Example

For the following graph, find

1. $f(0)$, $f(2)$, $f(3)$, $f(4)$.
2. Domain of f.
3. Range of f.
4. What is the x–intercept.

Exercise

For the following graph, find

1. $f(0)$ and $f(2)$.
2. Domain of f.
3. Range of f.
4. What is the x–intercept.

4 - Vertical and Horizontal line tests

1- Vertical Line Test:
Recall: A function is an assignment such that for each input, we have only one output.

Question: Given a graph (curve), how can we check if it is a graph of a function or not?

Answer: We apply the vertical line test (i.e., we draw vertical lines and we make sure it cuts the graph in at most one point, otherwise, the graph is not corresponding to a function).

Example

Test whether the following curves are graph of functions or not?

4 - Vertical and Horizontal line tests

2- Horizontal Line Test:

Question: Given a graph of a function, how can we check if the function has an inverse or not?

Answer: We apply the horizontal line test (i.e., we draw horizontal lines and we make sure it cuts the graph in at most one point, otherwise, the function has no inverse).

Example

Test whether the following functions have an inverse or not?

5 - Finding the graph of the inverse function from the graph of the original function

Recall: A function has an inverse if the graph of the function passes the horizontal line test.

Question: How can we find the graph of the inverse function from the graph of the original function?

Answer: We reflect the graph of the original function on the line $y = x$. The result is the graph of the original function.

Example

Sketch the graph of the inverse function for the following functions.