Section 3.2
Applications and Linear functions

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MATHS 103: Mathematics for Business I

Introduction

Recall: The price p of any item is related with the quantity q available of that item during either the supply or demand. i.e., the price is given as a function in the quantity

$$p = f(q).$$

Example

Suppose the demand per week for an item is 100 units when the price is 60 BD and 200 units when the price is 50 BD. Determine the demand function assuming it is a linear function.

Solution:
Let $p = f(q)$. Note that $f(100) = 60$ and $f(200) = 50$. So as a pair we have
$(\underbrace{100}_{x_1}, \underbrace{60}_{y_1})$ and $(\underbrace{200}_{x_2}, \underbrace{50}_{y_2})$.We need to find the slope which is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{50 - 60}{200 - 100} = \frac{-10}{100} = \frac{-1}{10}.$$

The equation of the linear function is given by

$$y - y_1 = m(x - x_1)$$

$$y - 60 = \frac{-1}{10}(x - 100)$$

$$y = \frac{-1}{10}x + 70 \rightarrow p = \frac{-1}{10}q + 70$$

Exercise

(Old Exam Question) A computer manufacturer will produce 2500 units when the price is 82 BD and 1900 units when the price is 80 BD. Find the supply equation assuming it is linear.

Exercise

Determine $f(x)$ if $f(x)$ is a linear function with the following
(1) $f(2) = 7$ and has slope 2.
(2) passes $f(1) = 2$ and $f(2) = 6$.

Example

Suppose that the cost to produce 10 units of a product is 40 BD and the cost of 20 units is 70 BD. If the cost c is linearly related to output q, find a linear equation relating c and q. Use that to find the cost to produce 35 units.

Solution: Let $c = f(q)$. Note that $f(10) = 40$ and $f(20) = 70$. So as a pair we have

$(\underbrace{10}_{x_1}, \underbrace{40}_{y_1})$ and $(\underbrace{20}_{x_2}, \underbrace{70}_{y_2})$.We need to find the slope which is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{70 - 40}{20 - 10} = \frac{30}{10} = 3.$$

The equation of the linear function is given by

$$y - y_1 = m(x - x_1)$$
$$y - 40 = 3(x - 10)$$
$$y = 3x + 10$$

Hence $c = 3q + 10$. The cost to produce 35 units is
$c = 3(35) + 10 = 115$

Example

(Inverse of Linear Functions)
(a) Does the linear function $f(x) = mx + b$ $(m \neq 0)$ has an inverse? Why? What is the name of the test?
(b) Find the inverse function and deduce that $f^{-1}(x)$ is again a linear function.

Solution: (b) To find the inverse, we follow the three steps of Section 2.4.
Step 0: Write $y = f(x)$.

$$y = mx + b$$

Step 1: Exchange x and y in step 0.

$$x = my + b$$

Step 2: Solve the literal equation in step 1 for y

$$x = my + b$$
$$x - b = my$$

Continue...

$$x = my + b$$
$$x - b = my$$
$$\frac{1}{m}x - \frac{b}{m} = y$$

Hence we have

$$f^{-1}(x) = \frac{1}{m}x - \frac{b}{m}$$

which is a linear function with slope $\frac{1}{m}$ and y-intercept $(0, \frac{-b}{m})$