Section 3.3
Quadratic functions

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MATHS 103: Mathematics for Business I

Introduction

Recall:

- A **quadratic function** is

$$f(x) = ax^2 + bx + c, \qquad a \neq 0$$

- The graph of a quadratic function is called **parabola**.
- The vertex is the point $(\frac{-b}{2a}, f(\frac{-b}{2a}))$.
- y-intercept is $(0, c)$.
- x-intercept is the solution of $ax^2 + bx + c = 0$. (use the formula in Section 0.8).
- The domain is $(-\infty, \infty)$.
- The range is either $[f(\frac{-b}{2a}), \infty)$ or $(-\infty, f(\frac{-b}{2a})]$ depending its open upward or downward.

Example

Sketch the graph of $y = x^2 + 4x - 12$.

Solution:

Here we have $a = 1, b = 4, c = -12$.

(1) Since $a = 1 > 0$, the parabola is upward.

(2) Vertex $= (\frac{-b}{2a}, f(\frac{-b}{2a})) = (\frac{-4}{-2}, f(\frac{-4}{2})) = (-2, f(-2)) = (-2, -16)$.

(3) y-intercept is $(0, -12)$.

(4) x-intercept: we solve $x^2 + 4x - 12 = 0$ to get

$$x = -6 \text{ or } x = 2 \qquad \text{using the formula in Section 0.8}$$

The x -intercept are the points

$$(-6, 0) \text{ or } (2, 0)$$

(5) Range $= [-16, \infty)$.

Exercise

Sketch the graph of $y = -x^2 + 6x - 5$.

Example

Sketch the graph of $y = x^2 + 4x + 4$.

Solution:

Here we have $a = 1, b = 4, c = 4$.

(1) Since $a = 1 > 0$, the parabola is upward.

(2) Vertex $= (\frac{-b}{2a}, f(\frac{-b}{2a})) = (\frac{-4}{2}, f(\frac{-4}{2})) = (-2, f(-2)) = (-2, 0)$.

(3) y-intercept is $(0, 4)$.

(4) x-intercept: we solve $x^2 + 4x + 4 = 0$ to get

$$x = -2 \qquad \text{using the formula in Section 0.8}$$

The x -intercept are the points

$$(-2, 0)$$

(5) Range $= [0, \infty)$.

Exercise

Sketch the graph of $y = x^2 + x + 1$.

Example

The demand function is $p = f(q) = 4 - 2q$, where p is the price and q is the number of units. Find the level of production that *maximize* the total revenue.

Solution:

$$\text{Total Revenue} = (\text{price per unit})(\text{number of units})$$
$$\text{Total Revenue} = (4 - 2q)(q)$$
$$\text{Total Revenue} = 4q - 2q^2$$

The maximum will be at the vertex, so we have

$$\text{Vertex} = \frac{-b}{2a} = \frac{-4}{2(-2)} = \frac{-4}{-4} = 1.$$

So the maximum is at $q = 1$ and $p = 4 - 2(1) = 2$.

Exercise

(Old Exam Question) The demand function for a product is $p = 80 - 2q$. Find the quantity that maximize the revenue.

Example

(Inverse of Quadratic Functions)
(a) Does the quadratic function $f(x) = ax^2 + bx + c$ $(a \neq 0)$ has an inverse? Why? What is the name of the test?
(b)Find the inverse function and deduce that $f^{-1}(x)$ is not a quadratic function.

Solution: (b) Let the domain be $[\frac{-b}{2a}, \infty)$. To find the inverse, we follow the three steps of Section 2.4.

Step 0: Write $y = f(x)$.

$$y = ax^2 + bx + c$$

Step 1: Exchange x and y in step 0.

$$x = ay^2 + by + c$$

Step 2: Solve the literal equation in step 1 for y

$$x = ay^2 + by + c$$

Continue...

$$x = ay^2 + by + x$$
$$0 = ay^2 + by + c - x$$

$$y = \frac{-b \pm \sqrt{b^2 - 4a(c - x)}}{2a}$$ By the formula in Section 0.8

So we take only one of them which is with the positive sign, so we have

$$f^{-1}(x) = \frac{-b + \sqrt{b^2 - 4a(c - x)}}{2a}$$

which is not a quadratic function!