Section 3.4
System of Linear Equations

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MATHS 103: Mathematics for Business I

Linear System

Definition

A **system of linear equations** in two variables x and y is a *list* of linear equations

$$\begin{aligned} a_1x + b_1y &= c_1 \\ a_2x + b_2y &= c_2 \\ &. \\ &. \\ &. \\ a_nx + b_ny &= c_n \end{aligned}$$

Goal: Find values for x and y such that all the equations above are true at the same time.

Geometry

- One way to solve such system is to graph each equation and the solution will be the point of intersection.

- There are two disadvantages of this way:
  1. We need to graph many equations accurately.
  2. It is hard to generalize it to linear systems of more than two variables or for non–linear systems.

Algebra

1- Elimination Method:

Example

Solve the following system:

$$x + 4y = 3 \quad (1)$$

$$-3x + 2y = -5 \quad (2)$$

Solution: We try to eliminate y first, so we multiple the Equation (1) by 2 and the Equation (2) by -4 to get

$$2x + 8y = 6 \quad (3)$$

$$12x - 8y = 20 \quad (4)$$

Now adding (3) and (4) yield $14x = 26$ and solving for x we get

$$14x = 26 \rightarrow x = \frac{26}{14} = \frac{13}{7}$$

Example

Solve the following system:

$$x + 4y = 3 \quad (1)$$
$$-3x + 2y = -5 \quad (2)$$

Now we substitute the value of x in either Equation (1) or (2) to find the missing y. We will substitute in Equation (1) to get

$$\frac{13}{7} + 4y = 3 \to 4y = 3 - \frac{13}{7}$$
$$4y = \frac{8}{7}$$
$$y = \frac{8}{28} = \frac{2}{7}$$

$$\text{Solution Set} = \{(\underbrace{\frac{13}{7}}_{x}, \underbrace{\frac{2}{7}}_{y})\}$$

Exercise

Solve the same example above, but eliminate x first.

$$x + 4y = 3 \quad (1)$$

$$-3x + 2y = -5 \quad (2)$$

Example

(Elimination Method) Solve the following system:

$$5x - 2y = 1 \tag{5}$$
$$3x + 3y = 9 \tag{6}$$

Solution: We try to eliminate x first, so we multiple the Equation (5) by 3 and the Equation (6) by -5 to get

$$15x - 6y = 3 \tag{7}$$
$$-15x - 15y = -45 \tag{8}$$

Now adding (7) and (8) yield $-21y = -42$ and solving for y we get

$$-21y = -42 \rightarrow y = \frac{-42}{-21} = 2$$

Example

Solve the following system:

$$5x - 2y = 1 \quad (5)$$

$$3x + 3y = 9 \quad (6)$$

Now we substitute the value of y in either Equation (5) or (6) to find the missing x. We will substitute in Equation (6) to get

$$3x + 3y = 9 \rightarrow 3x = 9 - 3(2)$$
$$3x = 3$$
$$x = \frac{3}{3} = 1$$

$$\text{Solution Set} = \{(\underbrace{1}_{x}, \underbrace{2}_{y})\}$$

Exercise

Solve the following system of equations:

$$7x - 4y = -4$$
$$2x - 5y = -5$$

2- Substitution Method:

This method will be more useful in solving non–linear systems in Section 3.5 and Section 3.6.

Example

(Substitution Method) Solve the following system:

$$2x - y = 1 \tag{9}$$
$$-x + 2y = 7 \tag{10}$$

Solution: Using Equation (9), solve for y (isolate y) in term of x to get

$$y = 2x - 1 \tag{11}$$

Now substitute y from Equation (11) into Equation (10) to get an equation in x only.

$$-x + 2(2x - 1) = 7$$

Continue...

$$
\begin{aligned}
-x + 2(2x-1) &= 7 \\
-x + 4x - 2 &= 7 \\
3x &= 9 \\
x &= 3
\end{aligned}
$$

Substitute back in Equation (11) ($y = 2x - 1$) we get

$$y = 2(3) - 1 = 5$$

$$\text{Solution Set} = \{(\underbrace{3}_{x}, \underbrace{5}_{y})\}$$

Exercise

Solve using the substitution method the following system of equations:

$$-x + 2y = 7$$
$$5x + 3y = -9$$