Section 3.5
System of Non–Linear Equations

Dr. Abdulla Eid

College of Science

MATHS 103: Mathematics for Business I

Linear System

Definition

A system of equations in which at least one equation *not* linear is called **non–linear system**

Strategy: We will solve non–linear system using the substitution method.

Geometry

- One way to solve such system is to graph each equation and the solution will be the point of intersection.

Dr. Abdulla Eid

- The main disadvantage of this way is that:
  1. We need to graph many equations accurately.

Algebra

Substitution Method:

Example

(Substitution Method) Solve the following system:

$$x^2 - 3 - y = 0 \tag{1}$$
$$2x + y = 5 \tag{2}$$

Solution: Using Equation (1), solve for y (isolate y) in term of x to get

$$y = x^2 - 3 \tag{3}$$

Now substitute y from Equation (3) into Equation (2) to get an equation in x only.

$$2x + (x^2 - 3) = 5$$

Continue...

$$2x + (x^2 - 3) = 5$$
$$x^2 + 2x - 3 = 5$$
$$x^2 + 2x - 8 = 0$$
$$x = 2 \text{ or } x = -4 \text{ by the Formula, (Section 0.8)}$$

Substitute back in Equation (3) ($y = x^2 - 3$) we get

$$y = 1 \text{ or } y = 13$$

$$\text{Solution Set} = \{(\underbrace{3}_{x}, \underbrace{5}_{y}), (\underbrace{-4}_{x}, \underbrace{13}_{y})\}$$

Solve the system.

$$y = x^3$$
$$x - 4y = 0$$

Example

(Substitution Method) Solve the following system:

$$y = 2\sqrt{x+1} \tag{4}$$
$$x = y + 2 \tag{5}$$

Solution: Using Equation (5), solve for x (isolate x) in term of y to get

$$x = y + 2 \tag{6}$$

Now substitute x from Equation (6) into Equation (4) to get an equation in y only.

$$y = 2\sqrt{y+2+1}$$

Continue...

$$y = 2\sqrt{y+2+1}$$
$$y = 2\sqrt{y+3}$$
$$y^2 = 4(y+3)$$
$$y^2 = 4y + 12$$
$$y^2 - 4y - 12 = 0$$
$$y = 2 \text{ or } y = -6 \text{ by the Formula, (Section 0.8)}$$

Substitute back in Equation (3) $(x = y + 2)$ we get

$$x = 4 \text{ or } x = -4(\textit{rejected!})$$

$$\text{Solution Set} = \{(\underbrace{4}_{x}, \underbrace{2}_{y})\}$$

Exercise

Solve the following system of equations:

$$y = \frac{4}{x}$$
$$2y = 3x + 1$$

Exercise

Solve the following system of equations:

$$y = \sqrt{x + 2}$$
$$x = y - 2$$