Section 3.6
Application of systems of Equations

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MATHS 103: Mathematics for Business I

Equilibrium Point

Definition

The **equilibrium point** is the point where the demand and supply meets.

Example

Find the equilibrium point if the supply and demand equations are $p = \frac{q}{40} + 10$ and $p = \frac{8000}{q}$, respectively.

Solution: We need to solve the system

$$p = \frac{q}{40} + 10 \tag{1}$$

$$p = \frac{8000}{q} \tag{2}$$

Substituting Equation (2) directly into Equation (1), we get

$$\frac{8000}{q} = \frac{q}{40} + 10$$

Continue...

$$\frac{8000}{q} = \frac{q}{40} + 10 \qquad \text{Multiply by } q$$

$$8000 = \frac{q^2}{40} + 10q \qquad \text{Multiply by } 40$$

$$32000 = q^2 + 400q$$

$$0 = q^2 + 400q - 32000$$

$$q = 400 \text{ or } q = -800(\textit{rejected}) \text{ by the Formula, (Section 0.8)}$$

Substitute back in Equation (2) ($p = \frac{8000}{q}$) we get

$$p = \frac{8000}{q} = \frac{8000}{400} = 20.$$

Example

(Old Exam Question) Find the equilibrium point if the supply and demand equations are $p = \sqrt{29 + 5q}$ and $p = 15 - q$, respectively. Moreover, find the revenue at the equilibrium point.

Solution: We need to solve the system

$$p = \sqrt{29 + 5q} \tag{3}$$

$$p = 15 - q \tag{4}$$

Substituting Equation (4) directly into Equation (3), we get

$$15 - q = \sqrt{29 + 5q}$$

Continue...

$$\begin{aligned} 15 - q &= \sqrt{29 + 5q} \\ (15 - q)^2 &= (\sqrt{29 + 5q})^2 \\ 225 - 30q + q^2 &= 29 + 5q \\ q^2 - 35q + 196 &= 0 \\ q = 28 \text{ or } q &= 7 \text{ by the Formula, (Section 0.8)} \end{aligned}$$

Substitute back in Equation (2) ($p = 15 - q$) we get

$$p = -13(\textit{rejected}) \text{ or } p = 8.$$

Hence $q = 8$ and $p = 7$. Total Revenue $= pq = 8(7) = 56$.

Example

(Break–even Points) A manufacturer sells a product at 4 BD per unit. If the fixed cost is 2000 BD and the variable cost is 2 BD per unit. Find the break–even point (i.e., the point where the total cost is equal to the total revenue (No profit)).

Solution: Recall that

$$\begin{aligned} \text{Total Revenue} &= (\text{price per unit})(\text{number of units sold}) = 4q \\ \text{Total cost} &= \text{fixed cost} + \text{variable cost} = 2000 + 2q \end{aligned}$$

In order to get the break–even point, we must have

$$\begin{aligned} \text{Total Revenue} &= \text{Total cost} \\ 4q &= 2000 + 2q \\ 2q &= 2000 \\ q &= 1000 \end{aligned}$$

So $p = 4(1000) = 4000$ BD.

Exercise

Find the break–even point if the total revenue is $3\sqrt{q}$ and the total cost is $2q + 500$.

Exercise

(Old Exam Question) Find the equilibrium point of

$$\text{Demand:} \quad 25q - 2p + 320 = 0$$

$$\text{Demand:} \quad 45q + p - 505 = 0$$

Exercise

(Old Final Exam Question) For a certain product, the material cost is 4 BDS per unit and the fixed cost is 50600 BD. If the price per unit is 6.5 BD. Find the total break–even point.