Section 4.2
Logarithmic Functions

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MATHS 103: Mathematics for Business I

1 - The Logarithmic Functions

Recall:

- The exponential function is

$$f(x) = a^x, \qquad a > 0, a \neq 1$$

- The general shape of $y = a^x$ is either

- Domain $=(-\infty, \infty)$.
- Range $= (0, \infty)$.

Question: Is $f(x)$ has an inverse? Why?

Answer: Yes, by the horizontal line test and the graph of the inverse function $f^{-1}(x)$ is either

- $f^{-1}(x)$ is called **logarithmic function** base a and it is denoted by

$$f^{-1}(x) = \log_a x$$

Note: (The fundamental equations)

1. $f(f^{-1})(x) = x$, so we have $a^{\log_a x} = x$.
2. $f^{-1}(f(x)) = x$, so we have $\log_a a^x = x$.

2 - Exponential and Logarithmic forms

We have the following

$$\underbrace{\log_a x = y}_{\text{logarithmic form}} \quad \text{if and only if} \quad \underbrace{x = a^y}_{\text{exponential form}}$$

Example

Convert from logarithmic form to exponential form and vice versa.

1. $3^2 = 9 \iff 2 = \log_3 9.$
2. $\log_2 1024 = 10 \iff 1024 = 2^{10}.$
3. $e^{-5} = y \iff -5 = \log_e y.$
4. $8^{\frac{2}{3}} = 4 \iff \frac{2}{3} = \log_8 4.$
5. $\log_2 \frac{1}{32} = -5 \iff \frac{1}{32} = 2^{-5}.$
6. $3^0 = 1 \iff 0 = \log_3 1.$

Exercise

Convert from the exponential form into logarithmic form and vice versa

1. $\log_7 x = 5$.
2. $\log_2 \sqrt{2} = \frac{1}{2}$.
3. $9^3 = 729$.
4. $5^{\frac{1}{3}} = \sqrt[3]{5}$.

Example

Solve for x the equation $\log_3 x = 4$.

Solution: We convert it into exponential form to get

$$x = 3^4 = 81$$

Solution set $= \{81\}$.

Example

Solve for x the equation $\log_x 4 = \frac{1}{2}$.

Solution: We convert it into exponential form to get

$$\begin{aligned} 4 &= x^{\frac{1}{2}} \\ 4^2 &= (x^{\frac{1}{2}})^2 \\ 16 &= x \end{aligned}$$

Solution set $= \{16\}$.

Example

Solve for x the equation $\log_4 x = -4$.

Solution: We convert it into exponential form to get

$$x = 4^{-4} = \frac{1}{256}$$

Solution set $= \{\frac{1}{256}\}$.

Exercise

Solve for x the equations

1. $\log_5 x = 3$.
2. $\log_3 1 = 0$.
3. $\log_a 1 = 0$.
4. $\log_x (2x + 8) = 2$.

Notation

- If $a = 10$, then we simply write $\log_{10}$ as $\log$ and it is called the **common logarithm**.
- If $a = e = 2.718281828\ldots$, then we simply write $\log_e$ as $\ln$ and it is called the **natural logarithm**.