Section 4.4
Logarithms and Exponential Equations

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MATHS 103: Mathematics for Business I

Type A - Logarithmic Equations

Strategy:

- Write the equation with single logarithm.
- Eliminate the logarithm and convert it to exponential form.
- Solve the resultant equation.

Example

Solve $\log x - \log 5 = \log 7$.

Solution:We write the equation with a single logarithm so we have

$$\log x - \log 5 - \log 7 = 0$$

$$\log \frac{x}{5 \cdot 7} = 0$$

$$\frac{x}{35} = 10^0 = 1 \rightarrow x = 35.$$

Solution Set$=\{35\}$.

Example

(Logarithmic Equation) Solve $\log_4(x-2)=1$.

Solution: Since we already have the equation with a single logarithm, we get rid of the logarithm by changing it to the exponential form.

$$\begin{aligned}\log_4(x-2)&=1\\(x-2)&=4^1\\x-2&=4\\x&=6\end{aligned}$$

Solution Set $=\{6\}$.

Exercise

Solve $\log_2(x-1) = 6$.

Example

(Logarithmic Equation) Solve $\log_2 x + \log_2(x-1) = 1$.

Solution: First we write it as single logarithm and then we get rid of the logarithm by changing it to the exponential form.

$$
\begin{aligned}
\log_2 x + \log_2(x-1) &= 1 \\
\log_2 x(x-1) &= 1 \\
x(x-1) &= 2^1 \\
x^2 - x &= 2 \\
x^2 - x - 2 &= 0
\end{aligned}
$$

$$x = 2 \text{ or } x = -1 \text{by the formula in Section 0.8}$$

We disregard (reject) $x = -1$ since we cannot have negative number inside the logarithm. so the only solution is $x = 2$. Solution Set $=\{2\}$.

Exercise

Solve $\log(x-3)+\log(x-5)=1$.

Example

(Logarithmic Equation) Solve $\log(x+2) - \log x = 2$.

Solution: We write it as single logarithm and then we get rid of the logarithm by changing it to the exponential form.

$$\begin{aligned}
\log(x+2) - \log x &= 2 \\
\log \frac{x+2}{x} &= 2 \\
\frac{x+2}{x} &= 10^2 \\
\frac{x+2}{x} &= 100 \\
x+2 &= 100x \\
2 &= 100x - x \\
2 &= 99x \\
x &= \frac{2}{99}
\end{aligned}$$

Exercise

Solve $\log(x+5) = \log(3x+2) + 1$.

Exercise

(Old Exam Question) Solve $\log_2 x + \log_2(x+2) = 3$.

Exercise

(Old Exam Question) Solve $\log 2 + \log(4 - x) = 2\log x$.

Exponential Equation

Example

Solve $\left(e^{3x-2}\right)^3 = e^3$.

Solution:

$$\begin{aligned}\left(e^{3x-2}\right)^3 &= e^3\\ e^{3(3x-2)} &= e^3\\ 3(3x-2) &= 3\\ 9x-6 &= 3\\ x &= 1\end{aligned}$$

Solution Set $=\{1\}$.

Exercise

(Old Exam Question) Solve the following equations:

1. $e^{\ln x + \ln 20} = 2x + 1$. (Hint: Use the fundamental equation)
2. $3^{2x-1} = 27$.

To solve exponential equations with different bases, we use the following strategy:

- We take the ln of both sides (in order to get rid of the exponent).
- We solve the resultant equation.

Example

Solve $(27)^{2x+1} = \frac{1}{3}$.

Solution: We take ln of both sides

$$\begin{aligned} \ln(27)^{2x+1} &= \ln\left(\frac{1}{3}\right) \\ (2x+1)\ln 27 &= \ln\left(\frac{1}{3}\right) \\ 2x+1 &= \frac{\ln\left(\frac{1}{3}\right)}{\ln 27} \\ 2x+1 &= \frac{-1}{3} \\ x &= \frac{-2}{3} \end{aligned}$$

Solution Set $=\{\frac{-2}{3}\}$.

Exercise

Solve $16^{x+1} = 4^{2x}$

Example

Solve $(10)^{\frac{x}{4}} = 6$.

Solution: We take ln of both sides

$$\begin{aligned} \ln(10)^{\frac{4}{x}} &= 6 \\ \left(\frac{4}{x}\right)\ln 10 &= \ln 6 \\ \frac{4}{x} &= \frac{\ln 6}{\ln 10} \\ 4\ln 10 &= x\ln 6 \\ x &= \frac{4\ln 10}{\ln 6} \end{aligned}$$

Solution Set $=\{\frac{4\ln 10}{\ln 6}\}$.

Exercise

Solve $2^{-4+\frac{3}{2}x} = 3$

Exercise

(Old Exam Question) Solve $3e^{2x-3} - 4 = 2$.

Example

Solve $(7)^{3x+2} = 8$.

Solution: We take ln of both sides

$$\begin{aligned}
\ln(7)^{3x+2} &= \ln 8 \\
(3x+2)\ln 7 &= \ln 8 \\
3x+2 &= \frac{\ln 8}{\ln 7} \\
3x &= \frac{\ln 8}{\ln 7} - 2 \\
x &= \frac{\frac{\ln 8}{\ln 7} - 2}{3}
\end{aligned}$$

Solution Set $=\{\frac{\frac{\ln 8}{\ln 7}-2}{3}\}$.