Section 5.1
Compound Interest

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MATHS 103: Mathematics for Business I

Recall: (Section 4.1) The compound interest formula is given by

$$A = P\left(1 + \frac{r}{m}\right)^{nm}$$

where,

- P = original (invested money) (**principal**).
- A = accumulated amount (future money).
- m = number of period per year to receive the interest.
- n = number of years that we are invested.
- r = annual interest rate which is called the **nominal rate** or **annual percentage rate (A.P.R)**.
- $I = A - P$ = accumulated interest.

(Note: You will need the material of Sections 4.2 and 4.4 for the following examples).

Example

How long it takes for 600 BD to amount to 800 BD at an annual rate of 4% compounded quarterly?

Solution:

$P = 600$, $A = 800$, $n = ?$, $r = 4\% = 0.04$, and $m = 4$. Thus

$$A = P\left(1 + \frac{r}{m}\right)^{nm}$$

$$800 = 600\left(1 + \frac{0.04}{4}\right)^{4n}$$

$$\frac{800}{600} = (1.01)^{4n}$$

$$\frac{4}{3} = 1.01^{4n}$$

$$\ln\frac{4}{3} = 4n\ln 1.01$$

$$\frac{\ln\frac{4}{3}}{\ln 1.01} = 4n \rightarrow \frac{\ln\frac{4}{3}}{4\ln 1.01} = n \rightarrow n \simeq 7.22$$

Exercise

Suppose 400 BD amounted to 580 BD in an saving account with interest rate of 3% compounded semi–annually. Find the number of years?

Example

Suppose 100 BD amounted to 160 BD in six years. If the interest was compounded quarterly, find the nominal rate that was earned by the money.

Solution:

$P = 100$, $A = 160$, $r = ?$, $n = 6$, and $m = 4$. Thus

$$A = P\left(1 + \frac{r}{m}\right)^{nm}$$

$$160 = 100\left(1 + \frac{r}{4}\right)^{4\cdot 6}$$

$$\frac{160}{100} = \left(1 + \frac{r}{4}\right)^{24}$$

$$1.6 = (1 + \frac{r}{4})^{24}$$

$$\ln 1.6 = 24\ln(1 + \frac{r}{4})$$

Continue...

$$\ln 1.6 = 24 \ln(1 + \frac{r}{4})$$

$$\frac{\ln 1.6}{24} = \ln(1 + \frac{r}{4})$$

$$0.0195834 = \ln(1 + \frac{r}{4})$$

$$e^{0.0195834} = (1 + \frac{r}{4})$$

$$1.019776499 = 1 + \frac{r}{4}$$

$$r = 0.0791$$

$$r = 7.9\%$$

Exercise

At what nominal rate of interest, compounded yearly, will 1 BD doubled in 10 years?

Example

The inflation rate in Bahrain for October 2015 is 2.75%. In how many years we will have to pay 2 BD to buy an item that we pay 1.6 BD now?

Solution:

$P = 1.6$, $A = 2$, $n = ?$, $r = 2.75\% = 0.0275$, and $m = 1$. Thus

$$A = P\left(1 + \frac{r}{m}\right)^{nm}$$

$$2 = 1.6\left(1 + \frac{0.0275}{1}\right)^{1n}$$

$$\frac{2}{1.6} = (1.0275)^{n}$$

$$\frac{2}{1.6} = 1.0275^{n}$$

$$\ln\frac{2}{1.6} = n\ln 1.0275$$

$$\frac{\ln\frac{2}{1.6}}{\ln 1.0275} \to n \simeq 8.22$$

Exercise

Same as the previous example with the inflation rate of 7% (as in 2008!) and for 1 BD to double.

Effective Rate

Example

An investor has a choice of investing a sum of money at 8% compounded annually or at 7.8% compounded semi–annually. Which is the better option?

Assume P BD is invested in an account that pays $r\%$ interest in m periods per year for one year. What will happen at the end of the year? We accumulate money and we get A. Now the rate of investing the P BD using the simple rate formula to get to A is called the **effective rate**. Thus we have

$$A_{\text{simple}} = A_{\text{compound}}$$
$$P + Pr_{\text{e}} = P(1 + \frac{r}{m})^m$$

Continue...

$$A_{\text{simple}} = A_{\text{compound}}$$

$$P + Pr_{\text{e}} = P(1 + \frac{r}{m})^m$$

$$Pr_{\text{e}} = P(1 + \frac{r}{m})^m - P$$

$$Pr_{\text{e}} = P((1 + \frac{r}{m})^m - 1)$$

$$r_{\text{e}} = (1 + \frac{r}{m})^m - 1$$

Example

What is the effective rate to a nominal rate of 4% compounded

1. Yearly:

$$r_e = (1 + \frac{r}{m})^m - 1 = (1 + \frac{0.04}{1})^1 - 1 = 1.04 - 1 = 0.04 = 4\%$$

2. semi–annually:

$$r_e = (1 + \frac{r}{m})^m - 1 = (1 + \frac{0.04}{2})^2 - 1 = 1.0404 - 1 = 0.0404 = 4.04\%$$

3. quarterly:

$$r_e = (1 + \frac{r}{m})^m - 1 = (1 + \frac{0.04}{4})^4 - 1 = 1.0406 - 1 = 0.0406 = 4.06\%$$

4. monthly:

$$r_e = (1 + \frac{r}{m})^m - 1 = (1 + \frac{0.04}{12})^{12} - 1 = 1.0407 - 1 = 0.0407 = 4.07\%$$

Exercise

Same as the previous example with nominal rate of 7%.

Example

An investor has a choice of investing a sum of money at 8% compounded annually or at 7.8% compounded semi–annually. Which is the better option?

Solution: We need to compare the effective rate of each one (which is the real rate in one year) and the larger will be the better option.

Option 1 Annually at 8%:

$$r_e = (1 + \frac{r}{m})^m - 1 = (1 + \frac{0.08}{1})^1 - 1 = 1.08 - 1 = 0.08 = 8\%$$

Option 2

semi–annually at 7.8%:

$$r_e = (1 + \frac{r}{m})^m - 1 = (1 + \frac{0.04}{2})^2 - 1 = 1.079521 - 1 = 0.0795$$

Thus, option 1 is better.

Exercise

An investor has a choice of investing a sum of money at 5% compounded daily or at 5.1% compounded quarterly. Which is the better option?