Section 5.3
Interest Compounded Continuously

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MATHS 103: Mathematics for Business I

Recall: (Section 4.1) The compound interest formula is given by

$$A = P\left(1 + \frac{r}{m}\right)^{nm},$$

where m is the number of times an interest is paid. Suppose the interest is paid *continuously* (*frequently*) (i.e., m is a large number ($m \to \infty$)). Then how much should we earn after n years?

Recall:

$$e = \lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{x}$$

So we have

$$A = \left(1 + \frac{r}{m}\right)^{mn} = P\left(1 + \frac{1}{\frac{m}{r}}\right)^{nm}$$

Set $x = \frac{m}{r}$, so we have $m = xr$ and we get

$$A = P\left(1 + \frac{1}{x}\right)^{nxr} = \ = P\left(\left(1 + \frac{1}{x}\right)^{x}\right)^{nr}$$

Take $x \to \infty$, we get

$$A = Pe^{nr}$$

Note: Think of the interest compounded continuously as an interest which will be paid in every second.
Effective rate:

$$r_e = e^r - 1$$

Present Value:

$$P = Ae^{-nr}$$

Example

If 1000 BD is deposited in a saving account that earns interest at an annual rate of 5.5% compounded continuously. What is the value of the account at the end of 3 years.

Solution:
$P = 1000$, $A = ?$, $n = 3$, and $r = 5.5\% = 0.055$. Thus

$$A = Pe^{nr} = 1000e^{3(0.055)} = 1179.39 \text{ BD}$$

Exercise

Find the compounded amount and the compounded interest if 400 BD is invested for 5 years in an account with interest compounded continuously

1. $5\frac{1}{4}\%$
2. 10%.

Exercise

Find the effective rate that corresponds to the given annual rate compounded continuously

1. 3%
2. 7%.
3. 2%
4. 10%.

Example

What is the annual rate compounded continuously corresponding to effective rate of 5%?

Solution:

$$
\begin{aligned}
r_e &= e^r - 1 \\
0.05 &= e^r - 1 \\
1.05 &= e^r \\
\ln 1.05 &= r \\
0.0488 &= r \\
4.88\% &= r
\end{aligned}
$$