University of Bahrain
Department of Mathematics
MATHS311: Abstract Algebra 1
Fall 2017
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Homework 10: Group Homomorphism
Due on December 21
Hand all the problems

Name: _____

1. Let $\varphi : G \to G'$ be a group homomorphism. Show the following:

   (a) If H' is a normal subgroup of G', then $\varphi^{-1}(H')$ is also a normal subgroup.

(b) If φ is an epimorphism and H is normal subgroup of G, then $\varphi(H)$ is a normal subgroup of G'.

(c) Give a counter example, where the result of part (c) does not hold if φ is **not** an epimorphism.

2. Let $\varphi : G \to G'$ be a group homomorphism with $\gcd(|G|, |G'|) = 1$. What are the possibilities of φ?

3. What is the kernel of the following homomorphism:

$$\begin{aligned} \varphi : \mathbb{Z}_{45} &\to \mathbb{Z}_{45} \\ x &\mapsto 5x \end{aligned}$$

4. Let $\varphi : G \to G$ be a group homomorphism. Define the set

$$F := \{a \in G \mid \varphi(a) = a\}$$

i.e., F is the set of elements in G that are fixed under φ. Prove that F is a (normal) subgroup of G.

5. Let $\varphi : G \to G'$ be an isomorphism. Prove the following:

   (a) $o(\varphi(a)) = o(a)$

   (b) $\varphi(Z(G)) = Z(G')$.

6. Find the left regular representation of $U(10)$ and $U(18)$. are the two groups isomorphic?