University of Bahrain
Department of Mathematics
MATHS311: Abstract Algebra 1
Fall 2017
Dr. Abdulla Eid

Homework 12: Isomorphism Theorems
Due on December 28
Hand all the problems

Name: _____

1. Show that the canonical (natural) map

$$\begin{aligned} \pi : G &\to G/H \\ g &\mapsto gH \end{aligned}$$

is a group homomorphism.

2. Let $\varphi : G \to G'$ be a group homomorphism. Show that $|\varphi(G)|$ divides both $|G|$ and $|G'|$.
(Hint: Use the first isomorphism theorem)

3. (For those who know about complex numbers)
   Prove that
$$\mathbb{Q}/\mathbb{Z} \simeq \{e^{2\pi i q} \mid q \in \mathbb{Q}\}$$

4. Find all subgroups of $\mathbb{Z}/10\mathbb{Z}$.

5. Let G be a finite group with $N \triangleleft G$. If $\tilde{H}$ is a subgroup of G/H, show that $\pi^{-1}(\tilde{H})$ is a subgroup of order $|H||N|$.
(Hint: Use the corresponding theorem and Lagrange's theorem)

6. Let $G = \mathbb{Z}_{24}$, $H = \langle 4 \rangle$ and $N = \langle 6 \rangle$.
(a) List the elements of HN (in additive notation, this is $H + N$) and $H \cap H$.

(b) List the cosets of HN/N and show the elements in each coset.

(c) List the cosets of $H/(H \cap N)$ and show the elements in each coset.

(d) Describe the corresponding between HN/N and $H/(H \cap N)$.

7. Prove the second isomorphism, i.e., prove

$$H/(H \cap N) \simeq HN/N$$