University of Bahrain
Department of Mathematics
MATHS311: Abstract Algebra 1
Fall 2017
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Homework 13: Direct Product
Due on January 4, 2018
Hand Problems 1 – 4

Name: _____

1. Write the element and the Cayley's table of $U(12) \times U(15)$.

2. Find the order of the following:

    1. $(7,7) \in U(12) \times U(15)$

    2. $(6, 15, 4) \in \mathbb{Z}_{30} \oplus \mathbb{Z}_{45} \oplus \mathbb{Z}_{24}$

3. Find an element of order 6 in $\mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_8$.

4. (a) Let $G = U(15)$, $H = \{1, 11\}$, $K = \{1, 2, 4, 8\}$. Show that G is an internal direct product of H and K.

(b) Let $G = S_3$, $H = \{(1), (123), (132)\}$, $K = \{(1), (12)\}$. Show that $S_3 = HK$ and $H \cap K = \{(1)\}$. Can you conclude that S_3 is an internal direct product of H and K?

5. Show that $G \times G' \simeq G' \times G$.

6. Show that $G \times G'$ is abelian if and only if G and G' are abelian.

7. Show that $\mathbb{Z}_2 \oplus \mathbb{Z}_8 \not\cong \mathbb{Z}_4 \oplus \mathbb{Z}_4$.

8. What are the possible orders of elements in $\mathbb{Z}_{65}, \mathbb{Z}_{72}, \mathbb{Z}_{100}$?