University of Bahrain
Department of Mathematics
MATHS311: Abstract Algebra 1
Fall 2017
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Homework 2: Elementary Properties of Groups and order of an element
Due on October 19
Hand on all the problems

Name: ______________________________

1. In a group $(G, *)$, prove that the equations $a * x = b$ and $y * c = d$ have unique solutions.

2. Let $(G, *)$ be a group such that $g^2 = e$, for all $g \in G$. Prove that $(G, *)$ is an abelian group.

   (Hint: Consider $(ab)^2$)

3. In class, we define a group to be a nonempty set with an associative binary operation $*$ such that

   Identity There exists an element e such that for all a we have

   $$a * e = a$$

   Inverse For all a, there exists a' such that

   $$a * a' = e$$

   Prove that in addition we get $e * a = a$ and $a' * a = e$. (Hence left-sided identity and inverse imply two-sided identity and inverse)

4. Let G be a group and $a \in G$ is an element of order n. Prove that if $n \mid k$, then $a^k = e$.

5. Translate each of the following expressions in additive notation:

   1. $(ab^2c^3)^{-3}$

   2. $(ab)^{-2}c^2 = e$

6. In a group G, If $a^{24} = e$, what are the possibilities for $o(a)$?

7. What is $o(6)$ in $(\mathbb{Z}_{14}, +_{14})$.

8. List all elements of order 10 in $(\mathbb{Z}_{10}, +_{10})$

9. In a group G, let $o(a) = 2000$, find $o(a^{185})$, $o(a^{400})$, $o(a^{7})$, and $o(a^{62})$

10. Let
$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 & -1 \\ 1 & -1 \end{pmatrix}$$
in $GL(2, \mathbb{R})$. Show that $o(A)$ and $o(B)$ are both finite, while $o(AB)$ is infinite.

11. 1. Prove that in a group, an element and it inverse have the same order.

2. For any $a, g \in G$, What is $(gag^{-1})^n$?

3. Prove that $o(gag^{-1}) = o(a)$.