University of Bahrain
Department of Mathematics
MATHS311: Abstract Algebra 1
Fall 2017
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Homework 3: Subgroups
Due on October 26
Hand on all the problems

Name: ______________________________

1. Show directly using Cayley's table that

$$\{0, 3, 6, 9\} \leq (\mathbb{Z}_{12}, +_{12})$$

2. Prove that

$$H = \{2^m \mid m \in \mathbb{Z}\} \leq (\mathbb{Q}^{\times}, \cdot)$$

3. Prove that
$$\mathrm{SL}(2,\mathbb{R}) := \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid ad - bc = 1, a, b, c, d \in \mathbb{R} \right\}$$
is a subgroup of $\mathrm{GL}(2,\mathbb{R})$ under matrix multiplication. (The group $\mathrm{SL}(2,\mathbb{R})$ is called the real *special linear group*)

4. Let G be a group, show that
$$H := \{g^2 \mid g \in G\}$$
is a subgroup.

5. Let $H, K \leq G$. Prove that

$$HK \leq G \iff HK = KH$$

In case G is abelian, can you conclude that $HK \leq G$?

6. Give an explicit example where the torsion subset is *not* a subgroup when G is non–abelian.
(Hint: Exercise 10 in Homework 2)

7. (i) Let $H = 2\mathbb{Z}$ and $K = 3\mathbb{Z}$ be two subgroups of $\mathbb{Z}$. Show that $H \cup K$ is **not** a subgroup of $\mathbb{Z}$.

   (Hint: Find one element in H and one element in K such that their sum is not in $H \cup K$)

   (ii) Let $H, K \leq G$. Prove that

$$H \cup K \leq G \iff H \subseteq K \text{ or } K \subseteq H$$

8. Let G be a group. Prove that
$$Z(G) \leq G$$