University of Bahrain
Department of Mathematics
MATHS311: Abstract Algebra 1
Fall 2017
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# Homework 4: Cyclic Groups
## Due on November 2
## Hand on all problems 1–5

Name: ____________________

1. Let $G = \mathrm{GL}(2, \mathbb{R})$ under matrix multiplication. Find the following:

   1. $C\left(\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}\right)$

   2. $C\left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right)$

   3. $Z(G)$

2. Let $G$ be a group.

   1. If $G$ is abelian, what are $Z(G)$ and $C(a)$? for any $a \in G$.

   2. Show that $C(a) = C(a^{-1})$ for all $a \in G$.

   3. Show that $Z(G) \leq C(a)$ for all $a \in G$.

   4. Show that $Z(G) = \cap_{a \in G} C(a)$.

3. Find $\langle 6, 8 \rangle$, $\langle 5, 3 \rangle$, and $\langle 6, 11 \rangle$ inside $(\mathbb{Z}, +)$. Can you conclude what is $\langle a, b \rangle$ if $\gcd(a, b) = 1$?

4. Find all subgroups of $(\mathbb{Z}_{45}, +_{45})$.

5. If $a, b \in G$ commute, show that

$$o(ab) \mid \mathrm{lcm}(o(a), o(b))$$

(Hint: Let $\ell = \mathrm{lcm}(o(a), o(b))$ and prove that $(ab)^\ell = e$)

6. 1. Show $n\mathbb{Z} \subseteq m\mathbb{Z}$ if and only if $n \mid m$.

2. Show $n\mathbb{Z} \cap m\mathbb{Z} = \{0\}$ if and only if $\gcd(n, m) = 1$.

7. Prove that $(\mathbb{Q}, +)$ is not finitely generated group.