University of Bahrain
Department of Mathematics
MATHS311: Abstract Algebra 1
Fall 2017
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Homework 5: Permutation Groups
Due on November 16
Hand on problems 1–6

Name: ______________________________

1. Let $\alpha := \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}, \beta := \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$. Show that $S_3 = \langle \alpha, \beta \rangle$.

2. For each of the following permutation, write it as disjoint product of cycles, find its order, determine whether it is odd or even permutation.

   1. $(123)(45)$ in S_5

   2. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 5 & 4 & 1 & 6 & 2 \end{pmatrix}$

   3. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 2 & 1 & 5 & 4 & 6 \end{pmatrix}$

   4. $(13256)(23)(46512)$ in S_6.

5. $(345)(245)$ in S_6.

6. $(13)(58)(2367)[(13)(58)]^{-1}$ in S_8.

3. Let $\alpha, \beta \in S_n$. Show that $\alpha^{-1}\beta^{-1}\alpha\beta$ is an even permutation.

4. Can you find an element in A_{12} of order 30?

5. Let $i \in \{1, 2, \ldots, n\}$. Define

$$\text{stab}(i) := \{\pi \in S_n \mid \pi(i) = i\}$$

Show that $\text{stab}(i)$ is a subgroup of S_n. It is called the *stabilizer of i in S_n* since it consists of those permutations that leaves i fixed. Can you find the oder of the stabilizer subgroup?

6. Let $D_4 := \{r_0, r_{90^\circ}, r_{180^\circ}, r_{270^\circ}, d_H, d_V, d_x, d_{-x}\}$. Write up the Cayley's table for D_4 and show it is not abelian.

7. (Important) Let $\tau = (a_1 a_2 \ldots a_k)$ be a cycle of length k.
(a) Prove that if σ is any permutation, then

$$\sigma\tau\sigma^{-1} = (\sigma(a_1)\,\sigma(a_2)\ldots\sigma(a_k))$$

is a cycle of length k.

(b) Let μ be a cycle of length k. Prove that there is a permutation σ such that $\sigma\tau\sigma^{-1} = \mu$.