University of Bahrain
Department of Mathematics
MATHS311: Abstract Algebra 1
Fall 2017
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Homework6: Cosets and Lagrange's Theorem
Due on November 16
Hand all the problems

Name: ________________________________

1. Find all the right and left cosets of the following subgroups and find the index $[G : H]$.

   1. $H := \{1, 17\}$ in $U(32)$.

   2. $H := \langle 3 \rangle$ in $U(32)$.

3. $H := \langle (1234) \rangle$ in S_4.

4. $H := \{r_0, r_{90^\circ}, r_{180^\circ}, r_{270^\circ}\}$ in D_4.

5. $H = \langle 12, 20 \rangle$ in $\mathbb{Z}_{40}$.

2. Let H be a subgroup of a group G. Prove that

$$a \in H \iff aH = a$$

3. Let H and K be two subgroups of a group G with $H \leq K \leq G$. Prove that

$$[G : H] = [G : K][K : H]$$

4. What fails in the proof that the number of left cosets and the number of right cosets is the same if we change the function to be $f(gH) = Hg$?

5. Prove that the order of $U(n)$ is even.
(Hint: What is the order of $n - 1$ in $U(n)$?)

6. Suppose that G is a group with more than one element and G has no proper, non-trivial subgroups. Prove that the order of G is finite.
(Do **not** assume that G is finite, rather prove that first!)

7. In this exercise, you will show that the converse of Lagrange's theorem is false.

   1. Write the converse of Lagrange's theorem.

   2. What is $|A_4|$?. Show that A_4 contains exactly eight 3-cycles by listing them all.

   3. Prove that is $\sigma \in A_n$, then $\pi\sigma\pi^{-1} \in A_n$.

   4. We will show that A_4 has **no** subgroup of order 6. Assume on the contrary that H is a subgroup of order 6. Prove that H must contain a 3-cycle.

   5. Let $\sigma \in H$ be a 3-cycle. Use Part (3) above and Exercise 7 in Homework 5 to show that H must contain all the 3-cycles and find the contradiction.