University of Bahrain
Department of Mathematics
MATHS311: Abstract Algebra 1
Fall 2017
Dr. Abdulla Eid

Homework 8: Quotient groups
Due on December 7
Hand all the problems

Name: ______________________________

1. For the groups in Question 1 in Homework 6, find the quotient group G/H if applicable.

2. Show that If G is a cyclic group, then G/H is a cyclic group.

3. Find the order of $9 + \langle 4 \rangle$ in $\mathbb{Z}_{12}$.

4. Let G be a group and let H be the torsion subgroup of G.

(a) Show that H is a normal subgroup.

(b) Show that every nonidentity element of G/H is of infinite order.

5. If $x^2 \in H$ for all $x \in G$, prove that H is a normal subgroup of G and that G/H is abelian group.

6. (Important) Let G be a group and H is a subgroup of G. Define the **normalizer** of H in G, denoted by $N_G(H)$ by

$$N_G(H) := \{a \in G \mid aH = Ha\}$$

(a) Prove that $N_G(H)$ is a subgroup of G.

(b) Let $G = S_4$ and $H = \text{stab}(2)$. What is $N_G(H)$?

(c) Show that H is normal in $N_G(H)$.

(d) Show that if H is normal in a subgroup K, then $K \subseteq N_G(H)$.
(i.e., $N_G(H)$ is the largest subgroup where H is a normal subgroup of).

(e) If H is normal subgroup of G, what is $N_G(H)$?

7. (Important) Let G be a group and let $G' = \langle aba^{-1}b^{-1} \rangle$, i.e., that is, G' is the subgroup of all finite products of elements in G of the form $aba^{-1}b^{-1}$.

   (a) Show that G' is a subgroup of G. (G' is called the commutator subgroup of G)

   (b) Show that G' is a normal subgroup of G.

(c) Show that G/G' is an abelian group.

(d) Let H be a normal subgroup of G. Show that G/H is abelian if and only if H contains G'.