

University of Bahrain
 Department of Mathematics
 MATHS311: Abstract Algebra 1
 Fall 2017
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Project 1: Maps from \mathbb{Z}_n to \mathbb{Z}_m

The aim of this project is to provide further practice in:

1. The groups $(\mathbb{Z}_n, +_n)$
2. Order of an element in a group
3. Generators of a cyclic group
4. Group homomorphisms

Let $\varphi : \mathbb{Z}_n \rightarrow \mathbb{Z}_m$ be a group homomorphism. In this project, we would like to classify completely all such maps.

1. Show that φ is completely determined by the value of $\varphi(1)$.
2. Show that $o(\varphi(1))$ divides $\gcd(n, m)$.
3. Show that the map $\varphi : \mathbb{Z}_{18} \rightarrow \mathbb{Z}_{30}$ given by $\varphi(a) = 14a$ is **not** a group homomorphism.
4. Find the condition where the identity map $\varphi : \mathbb{Z}_n \rightarrow \mathbb{Z}_m$ is a group homomorphism.
5. If $b \in \mathbb{Z}_m$ with $o(b)$ divides $\gcd(n, m)$, then there exists a group homomorphism $\varphi_b : \mathbb{Z}_n \rightarrow \mathbb{Z}_m$ such that $\varphi_b(1) = b$.
6. Given the fact that the number of group homomorphisms $\varphi : \mathbb{Z}_n \rightarrow \mathbb{Z}_m$ is equal to $\gcd(n, m)$. Find all the group homomorphisms $\varphi : \mathbb{Z}_{18} \rightarrow \mathbb{Z}_{30}$.
7. Find the kernel and the image of two homomorphisms above.