University of Bahrain
Department of Mathematics
MATHS311: Abstract Algebra 1
Fall 2017
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Project 2: Dihedral Groups

The aim of this project is to provide further practice in:

1. Rigid Motion
2. Group of Symmetries
3. Dihedral Groups
4. Representation of a group

A **symmetry** of a geometric figure is a rearrangement of the figure preserving the arrangement of its sides and vertices as well as its distances and angles. A map from the plane to itself preserving the symmetry of an object is called a **rigid motion**.

We have seen the group of symmetries of the square D_4 in class. Anagously we can define the nth **dihedral group** D_n to be the group of all rigid motions of the plane of regular n–gon. The group D_n consists of rotations and reflections.

1. What is the group of symmetries of an equilateral triangle D_3? (Describe the elements as well as the Cayley's table)

   (Hint: D_3 is the symmetry group of the Mercedes–Benz logo)

2. What is the size of D_n? (Hint: How many choices we can replace the first vertex? If the first vertex is fixed, how many choices we have for the second one? Try $= 3, 4, 5$ to get an idea).

3. How many different rotations in D_n?

4. Show that the subgroup of all rotations R_n is a cyclic group generated by a rotation r. What is the order of the rotation subgroup?

5. Show that any reflection has order 2.

6. Let s be the reflection that fixes vertex 1. Show that $srs = r^{-1}$. conclude that D_n is nonabelian group.

7. Show that R_n is a normal subgroup of D_n and that D_n/R_n is a group of order 2.

8. Show that D_n is generated by the elements r, s subject to the conditions $r^n = e, s^2 = e, srs = r^{-1}$.