University of Bahrain
Department of Mathematics
MATHS311: Abstract Algebra 1
Fall 2017
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Project 3: Quaternion Groups

The aim of this project is to provide further practice in:

1. Cayley's table.
2. Order of an element.
3. Subgroups and normal subgroups
4. Quaternion group
5. Isomorphic groups

It was invented by William Hamilton in 1843. The quaternions are used to describe rotations in three dimensional space, and they are used in physics. The quaternions can be used to extend the complex numbers in a natural way. Consider the set

$$Q_8 = \{1, \mathbf{i}, \mathbf{j}, \mathbf{k}, -1, -\mathbf{i}, -\mathbf{j}, -\mathbf{k}\}$$

together with the operation

$$-1 \cdot -1 = 1, \mathbf{i}^2 = \mathbf{j}^2 = \mathbf{j}^2 = -1, \mathbf{ij} = \mathbf{k}, \mathbf{jk} = \mathbf{i}, \mathbf{ki} = \mathbf{j}, \mathbf{ji} = -\mathbf{j}, \mathbf{ik} = -\mathbf{j}, \mathbf{kj} = -\mathbf{i}$$

1. Construct the Cayley's table for Q_8. Is it an abelian group?

2. Find the order of every element in Q_8.

3. List all the cyclic subgroups of the quaternions.

4. List all the subgroups of the quaternions.

5. What is $Z(Q_8)$?

6. Show that every subgroup of the quaternions is normal.

7. Let $H = \{-1, 1\}$, construct the Cayley's table for G/H. Is it isomorphic to $\mathbb{Z}_4$?

8. Prove that Q_8 is **not** isomorphic to the dihedral group D_4.