Section 5.2
Diagonalization

Dr. Abdulla Eid

College of Science

MATHS 211: Linear Algebra

Goal:

1. Finding diagonalization of a matrix.
2. When has a matrix A, a diagonalization?
3. Benefits of diagonalization of a matrix.

Definition 1

If A is an $n \times n$ matrix, then a **nonzero** vector $\mathbf{x}$ in $\mathbb{R}^n$ is called an **Eigenvector** of A if

$$A\mathbf{x} = \lambda \mathbf{x}$$

for some scalar $\lambda \in \mathbb{R}$. The scalar λ is an **Eigenvalue** of A and $\mathbf{x}$ is said to be the **Eigenvector** corresponding to λ.

Characteristic Polynomial of a matrix

Theorem 2

If A is an $n \times n$ matrix, then λ is an Eigenvalue if and only if

$$\det(\lambda I_n - A) = \mathbf{0}$$

This is called the characteristic polynomial of A.

Example 3

Find the Diagonalization of

$$A = \begin{pmatrix} 2 & -1 \\ 10 & -9 \end{pmatrix}$$

Example 4

Write the following matrix

$$A = \begin{pmatrix} 3 & 0 \\ 5 & 3 \end{pmatrix}$$

as $A = PDP^{-1}$, for some matrix P and diagonal matrix D.

Questions: How can we do that?
When that can happen?
Why would you that in the first place?

Example 5

Write the following matrix

$$A = \begin{pmatrix} -2 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

as $A = PDP^{-1}$, for some matrix P and diagonal matrix D.

Questions: How can we do that?

When that can happen?

Why would you that in the first place?

Example 6

Write the following matrix

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 0 & 1 & 3 \end{pmatrix}$$

as $A = PDP^{-1}$, for some matrix P and diagonal matrix D.

Questions: How can we do that?

When that can happen?

Why would you that in the first place?

When can we diagonalize a matrix?

Theorem 7

A is diagonalizable if and only if A has exactly n linearly independent Eigenvectors.

A shortcut (sometimes is useful)

Theorem 8

If A has n distinct Eigenvalues, then A is diagonalizable.

Why diagonalization?

Example 9

Find A^{11}, where

$$A = \begin{pmatrix} -1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & -3 & 1 \end{pmatrix}$$

Why diagonalization?

Example 10

Find A^{1000}, A^{-1000}, A^{2017}, A^{20}, where

$$A = \begin{pmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Why diagonalization?

If A is diagonalizable, i.e., $A = PDP^{-1}$, then we have

1. $A^{-1} = PD^{-1}P^{-1}$.
2. $A^n = PD^nP^{-1}$.
3. $\det(A) = \det(D) =$ multiplication of the Eigenvalues.
4. $Rank(A) = Rank(PDP^{-1})$.
5. $Nullity(A) = Nullity(PDP^{-1})$.
6. $Trace(A) = Trace(PDP^{-1})$.