Section 4.1
Relative Extrema
3 Lectures

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MATHS 101: Calculus I

First Derivative Test

Question: How to find the local minimum and local maximum?

Theorem 1

(First Derivative Test)

1. *If $f'(x)$ changes from positive to negative as x increases, then f has a local maximum at a.*
2. *If $f'(x)$ changes from negative to positive as x increases, then f has a local minimum at a.*

Example 2

Find the intervals where the function is increasing/decreasing and find all local max/min.

$$f(x) = 2x^3 + 3x^2 - 36x$$

Solution:

We find the derivative first which is

$$f'(x) = 6x^2 + 6x - 36$$

To find the critical points, we find where the derivative equal to zero or does not exist.

$$f'(x) = 0$$

$$\text{numerator} = 0$$

$$6x^2 + 6x - 36 = 0$$

$$x = 2 \text{ or } x = -3$$

$$f'(x) \text{ does not exist}$$

$$\text{denominator} = 0$$

$$1 = 0$$

Always False

No Solution

Number Line

1. f is increasing in $(-\infty, -3) \cup (2, \infty)$.
2. f is decreasing in $(-3, 2)$.
3. f has a local maximum at $x = -3$ with value $f(-3) = 66$.
4. f has a local minimum at $x = 2$ with value $f(2) = -44$.

Exercise 3

Find the intervals where the function is increasing/decreasing and find all local max/min.

$$f(x) = x^4 - 2x^2 + 3$$

Example 4

Find the intervals where the function is increasing/decreasing and find all local max/min.

$$f(x) = \sin x + \cos x, \qquad x \in [0, 2\pi]$$

Solution:
We find the derivative first which is

$$f'(x) = \cos x - \sin x$$

$$f'(x) = 0$$
$$\text{numerator} = 0$$
$$\sin x - \cos x = 0$$
$$\sin x = \cos x$$
$$\tan x = 1 \rightarrow x = \tan^{-1} 1$$
$$x = \frac{\pi}{4} \text{ or } x = \frac{5\pi}{4}$$

$$f'(x) \text{ does not exist}$$
$$\text{denominator} = 0$$
$$1 = 0$$

Always False

No Solution

Number Line

1. f is increasing in $(0, \frac{\pi}{4}) \cup (\frac{5\pi}{4}, 2\pi)$.
2. f is decreasing in $(\frac{\pi}{4}, \frac{5\pi}{4})$.
3. f has a local maximum at $x = \frac{\pi}{4}$ with value $f(\frac{\pi}{4}) = \sqrt{2}$.
4. f has a local minimum at $x = \frac{5\pi}{4}$ with value $f(\frac{5\pi}{4}) = -\sqrt{2}$.

Exercise 5

Find the intervals where the function is increasing/decreasing and find all local max/min.

$$f(x) = x^2 - x - \ln x$$

Example 6

Find the intervals where the function is increasing/decreasing and find all local max/min.

$$f(x) = \frac{x^2}{x-1}$$

Solution:

We find the derivative first which is

$$f'(x) = \frac{(x-1)(2x) - (x^2)(1)}{(x-1)^2} = \frac{x^2 - 2x}{(x-1)^2}$$

To find the critical points, we find where the derivative equal to zero or does not exist.

$$f'(x) = 0$$
$$\text{numerator} = 0$$
$$x^2 - 2x = 0$$
$$x = 0 \text{ or } x = 2$$

$$f'(x) \text{ does not exist}$$
$$\text{denominator} = 0$$
$$(x-1)^2 = 0$$
$$x = 1$$

Number Line

1. f is increasing in $(-\infty, 0) \cup (2, \infty)$.
2. f is decreasing in $(0, 2)$.
3. f has a local maximum at $x = 0$ with value $f(0) = 0$.
4. f has a local minimum at $x = 2$ with value $f(2) = 4$.

Exercise 7

Find the intervals where the function is increasing/decreasing and find all local max/min.

$$f(x) = \sin^{-1} x$$

Solution:

We find the derivative first which is

$$f'(x) = \frac{1}{\sqrt{1-x^2}}$$

To find the critical points, we find where the derivative equal to zero or does not exist.

$f'(x)$ does not exist

$$\text{numerator} = 0$$
$$1 = 0$$

Always False

No Solution

$f'(x) = 0$

$$\text{denominator} = 0$$
$$\sqrt{1-x^2} = 0$$
$$x = -1 \text{ or } x = 1$$

Number Line

Recall the domain of $f(x) = \sin^{-1} x$ which is $[-1, 1]$.

1. f is increasing in $(-1, 1)$.
2. f has no local maximum nor local minimum.

Example 8

If f is an increasing function. Show that f^{-1} is an increasing function.

Solution:

Since f is an increasing function, then $f' > 0$. Now we find the derivative of f^{-1} which is

$$\left(f^{-1}(y)\right)' = \frac{1}{f'(f^{-1}(y))} > 0$$

(Critical point, but not local extreme)

Exercise 9

Show that 3 is a critical number for $f(x) = 7 + (x-3)^5$, but f does not have a local extreme at $x = 3$.