Section 1.3 (Part 2)
Addition

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MATHS 211: Linear Algebra

1- Addition and Scalar Multiplication

Definition 1

Let $A = (A_{ij})$, $B = (B_{ij})$ be two matrices of the same size, and $c \in \mathbb{R}$ is a real number.

- Matrix addition

$$A + B = (A_{ij} + B_{ij}) \quad \text{“adding coordinatewise“}$$

- Scalar Multiplicaion

$$cA = (cA_{ij}) \quad \text{“multipy everything by“} c$$

Example 2

Let

$$A = \begin{pmatrix} -2 & 1 \\ 2 & -3 \end{pmatrix}, \quad B = \begin{pmatrix} -5 & -5 \\ 3 & -3 \end{pmatrix}, \quad C = \begin{pmatrix} -1 & -1 \\ 3 & -3 \end{pmatrix}, \quad \mathbf{0} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Fine:

1.
$$A + B = \begin{pmatrix} -2 & 1 \\ 2 & -3 \end{pmatrix} + \begin{pmatrix} -5 & -5 \\ 3 & -3 \end{pmatrix} = \begin{pmatrix} -2-5 & 1-5 \\ 2+3 & -3-3 \end{pmatrix}$$
$$= \begin{pmatrix} -7 & -4 \\ 5 & -6 \end{pmatrix}$$

2.
$$2B = 2\begin{pmatrix} -5 & -5 \\ 3 & -3 \end{pmatrix} = \begin{pmatrix} 2(-5) & 2(-5) \\ 2(3) & 2(-3) \end{pmatrix}$$
$$= \begin{pmatrix} -10 & -10 \\ 6 & -6 \end{pmatrix}$$

Example 3

Let

$$A = \begin{pmatrix} -2 & 1 \\ 2 & -3 \end{pmatrix}, \quad B = \begin{pmatrix} -5 & -5 \\ 3 & -3 \end{pmatrix}, \quad C = \begin{pmatrix} -1 & -1 \\ 3 & -3 \end{pmatrix}, \quad \mathbf{0} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Find:

$$\begin{aligned} 2A - 3B &= 2\begin{pmatrix} -2 & 1 \\ 2 & -3 \end{pmatrix} - 3\begin{pmatrix} -5 & -5 \\ 3 & -3 \end{pmatrix} \\ &= \begin{pmatrix} -4 & 2 \\ 4 & -6 \end{pmatrix} - \begin{pmatrix} -15 & -15 \\ 9 & -9 \end{pmatrix} \\ &= \begin{pmatrix} 11 & 17 \\ -5 & 3 \end{pmatrix} \end{aligned}$$

Example 4

Let

$$A = \begin{pmatrix} -2 & 1 \\ 2 & -3 \end{pmatrix}, \quad B = \begin{pmatrix} -5 & -5 \\ 3 & -3 \end{pmatrix}, \quad C = \begin{pmatrix} -1 & -1 \\ 3 & -3 \end{pmatrix}, \quad \mathbf{0} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Find:

$$\begin{aligned} 3A + C^T &= 3\begin{pmatrix} -2 & 1 \\ 2 & -3 \end{pmatrix} + \begin{pmatrix} -5 & -5 \\ 3 & -3 \end{pmatrix}^T \\ &= \begin{pmatrix} -6 & 3 \\ 6 & -9 \end{pmatrix} + \begin{pmatrix} -1 & 3 \\ -3 & -3 \end{pmatrix} \\ &= \begin{pmatrix} -7 & 6 \\ 3 & -12 \end{pmatrix} \end{aligned}$$

Example 5

Let

$$A = \begin{pmatrix} -2 & 1 \\ 2 & -3 \end{pmatrix}, \quad B = \begin{pmatrix} -5 & -5 \\ 3 & -3 \end{pmatrix}, \quad C = \begin{pmatrix} -1 & -1 \\ 3 & -3 \end{pmatrix}, \quad \mathbf{0} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Find:

$$\begin{aligned} (2A - B)^T &= \left(2 \begin{pmatrix} -2 & 1 \\ 2 & -3 \end{pmatrix} - \begin{pmatrix} -5 & -5 \\ 3 & -3 \end{pmatrix} \right)^T \\ &= \left(\begin{pmatrix} -4 & 2 \\ 4 & -6 \end{pmatrix} - \begin{pmatrix} -5 & -5 \\ 3 & -3 \end{pmatrix} \right)^T \\ &= \begin{pmatrix} 1 & 7 \\ 1 & 9 \end{pmatrix}^T = \begin{pmatrix} 1 & 1 \\ 7 & 9 \end{pmatrix} \end{aligned}$$

Exercise 6

Find $A + \mathbf{0}$, $\mathbf{0} + A$. What do you conclude? What it is the name of $\mathbf{0}$?

Exercise 7

(Old Exam Question) Let

$$A = \begin{pmatrix} -3 & 1 & 5 \\ 2 & 1 & 0 \end{pmatrix} \qquad C = \begin{pmatrix} 2 & 5 \\ 6 & 3 \\ 0 & -4 \end{pmatrix}$$

Find $3A - 2C^T$ and $2A + \mathbf{0}$

Example 8

Solve

$$\begin{pmatrix}4\\6\\8\end{pmatrix} + 2\begin{pmatrix}x\\y\\2z\end{pmatrix} = \begin{pmatrix}-10\\-24\\20\end{pmatrix}$$

$$\begin{pmatrix}4\\6\\8\end{pmatrix} + 2\begin{pmatrix}x\\y\\2z\end{pmatrix} = \begin{pmatrix}-10\\-24\\20\end{pmatrix}$$

$$\begin{pmatrix}4\\6\\8\end{pmatrix} + \begin{pmatrix}2x\\2y\\4z\end{pmatrix} = \begin{pmatrix}-10\\-24\\20\end{pmatrix}$$

continue...

$$\begin{pmatrix} 4+2x \\ 6+2y \\ 8+4z \end{pmatrix} = \begin{pmatrix} -10 \\ -24 \\ 20 \end{pmatrix}$$

$$4+2x = 10 \to x = 3$$

$$6+2y = -24 \to y = -15$$

$$8+4z = 20 \to z = 5$$

Solution Set$=\{(3,-15,5)\}$.

Vector Space Axioms

Consider the set $\text{Mat}(m, n, \mathbb{R})$ of $m \times n$ matrices together with the operations:

$$\begin{aligned} + : \text{Mat}(m, n, \mathbb{R}) \times \text{Mat}(m, n, \mathbb{R}) &\to \text{Mat}(m, n, \mathbb{R}) \\ (A, B) &\mapsto A + B \end{aligned}$$

and

$$\begin{aligned} \cdot : \mathbb{R} \times \text{Mat}(m, n, \mathbb{R}) &\to \text{Mat}(m, n, \mathbb{R}) \\ (k, B) &\mapsto kB \end{aligned}$$

Satisfies the following properties:

A1: Closure For any $A, B \in \text{Mat}(m, n, \mathbb{R})$, we have

$$A + B \in \text{Mat}(m, n, \mathbb{R})$$

Vector Space Axioms

A1: Closure For any $A, B \in \text{Mat}(m, n, \mathbb{R})$, we have

$$A + B \in \text{Mat}(m, n, \mathbb{R})$$

A2: Associativity For any $A, B, C \in \text{Mat}(m, n, \mathbb{R})$, we have

$$(A + B) + C = A + (B + C)$$

A3: Zero There exists a special matrix $\mathbf{0} \in \text{Mat}(m, n, \mathbb{R})$ such that

$$A + \mathbf{0} = A = \mathbf{0} + A$$

A4: Negative There is an element $-A \in \text{Mat}(m, n, \mathbb{R})$ such that

$$A + (-A) = \mathbf{0} = (-A) + A$$

A2: Commutative For any $A, B \in \text{Mat}(m, n, \mathbb{R})$, $A + B = B + A$

Vector Space Axioms

S1: Closure For any $k, \ell \in \mathbb{R}$ and $A, B \in \text{Mat}(m, n, \mathbb{R})$, we have

$$k \cdot A \in \text{Mat}(m, n, \mathbb{R})$$

S2:

$$k \cdot (A + B) = k \cdot A + k \cdot B$$

S3:

$$(k + \ell) \cdot A = k \cdot A + \ell \cdot A$$

S4:

$$(k\ell) \cdot A = k(\ell \cdot A)$$

S5:

$$1 \cdot A = A$$