

# Section 2.1

## Determinant of a matrix

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MATHS 211: Linear Algebra

## Goal:

- 1 To define the determinant of a matrix.
- 2 To find the determinant of a matrix using cofactor expansion (Section 2.1).
- 3 To find the determinant of a matrix using row reduction (Section 2.2).
- 4 Explore the properties of the determinant and its relation to the inverse. (Section 2.3)
- 5 To solve linear system using the Cramer's rule. (Section 2.3)

# 1 - Definition of the determinant of a matrix

## Example 1

(Motivational Example) Find the inverse of the following  $2 \times 2$  matrix and state the condition that it will exist.

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

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## Determinant of $2 \times 2$ matrix

### Definition 2

The number  $ad - bc$  is called the *determinant* of  $A$

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### Example 3

Let

$$A = \begin{pmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ 2 & 0 & 1 \end{pmatrix}$$

Find all minors and cofactors.

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## Example 4

Find  $\det(A)$  for

$$A = \begin{pmatrix} 1 & 2 & 4 \\ -3 & 3 & 5 \\ 7 & 0 & 6 \end{pmatrix}$$

Solution:

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# General Definition of the determinant

## Definition 5

The determinant of a matrix is a *signed* sum of the product of exactly one element from each row provided that we do not use the same column twice.

## Definition 6

Let  $A$  be a square matrix.

- 1 **The minor of entry**  $a_{ij}$  is denoted by  $M_{ij}$  and is defined to be the determinant of the submatrix that remains after the  $i$ th row and  $j$ th column are deleted from  $A$ .
- 2 **The cofactor entry of entry**  $a_{ij}$  is denoted by  $C_{ij}$  and equals  $(-1)^{i+j}M_{ij}$ .

Note that  $C_{ij} = \pm M_{ij}$  depending on the “checkerboard”:

$$\begin{pmatrix} + & - & + & - & + & \dots \\ - & + & - & + & - & \dots \\ + & - & + & - & + & \dots \\ \dots & \dots & \dots & \dots & & \dots \end{pmatrix}$$



## Definition 7

Let  $A$  be a square matrix, **determinant of  $A$** , denoted by  $\det(A)$  is sum of the entries in any row or column multiplied by their cofactors, i.e.,

$$\det(A) = a_{1j}C_{1j} + a_{2j}C_{2j} + \cdots + a_{nj}C_{nj} \quad \text{column-wise}$$

or

$$\det(A) = a_{i1}C_{i1} + a_{i2}C_{i2} + \cdots + a_{in}C_{in} \quad \text{row-wise}$$

## Example 8

Find  $\det(A)$  for

$$A = \begin{pmatrix} 5 & 2 & -2 & 0 \\ 3 & 2 & -2 & 0 \\ 1 & 0 & -1 & 1 \\ 0 & -1 & 5 & 7 \end{pmatrix}$$

Solution:

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### Example 9

Find the determinant for following matrices

$$A = \begin{pmatrix} 2 & -12 & 14 \\ 0 & 3 & 50 \\ 0 & 0 & 6 \end{pmatrix}, B = \begin{pmatrix} 5 & 0 & 0 \\ -3 & -3 & 0 \\ 7 & 0 & 2 \end{pmatrix}, C = \begin{pmatrix} 6 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Solution:

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## Theorem 10

*If  $A$  is an  $n \times n$  triangular matrix (upper triangular, lower triangular, or diagonal), then  $\det(A)$  is the product of the entries on the main diagonal of the matrix, that is  $\det(A) = a_{11}a_{22} \dots a_{nn}$ .*

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